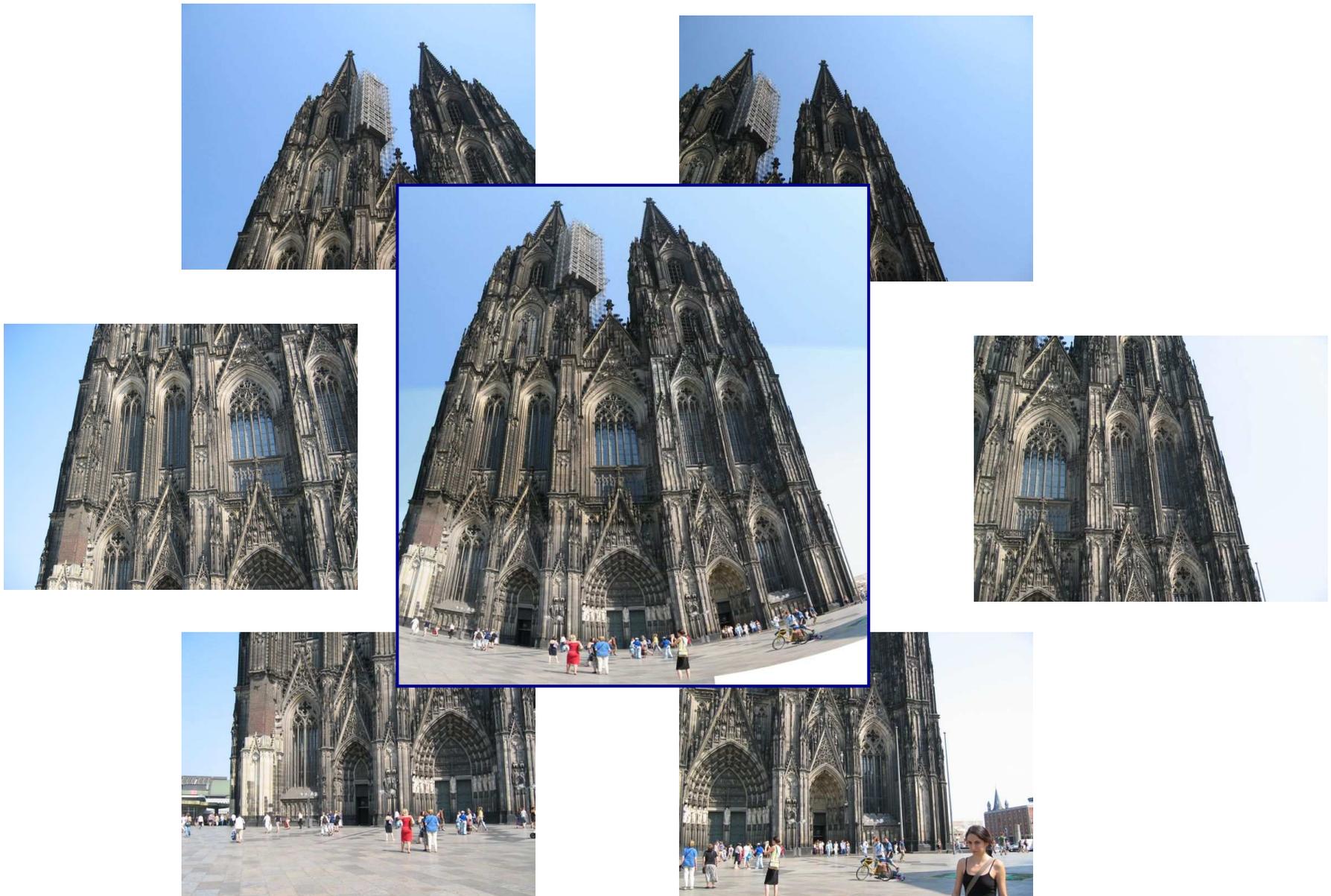


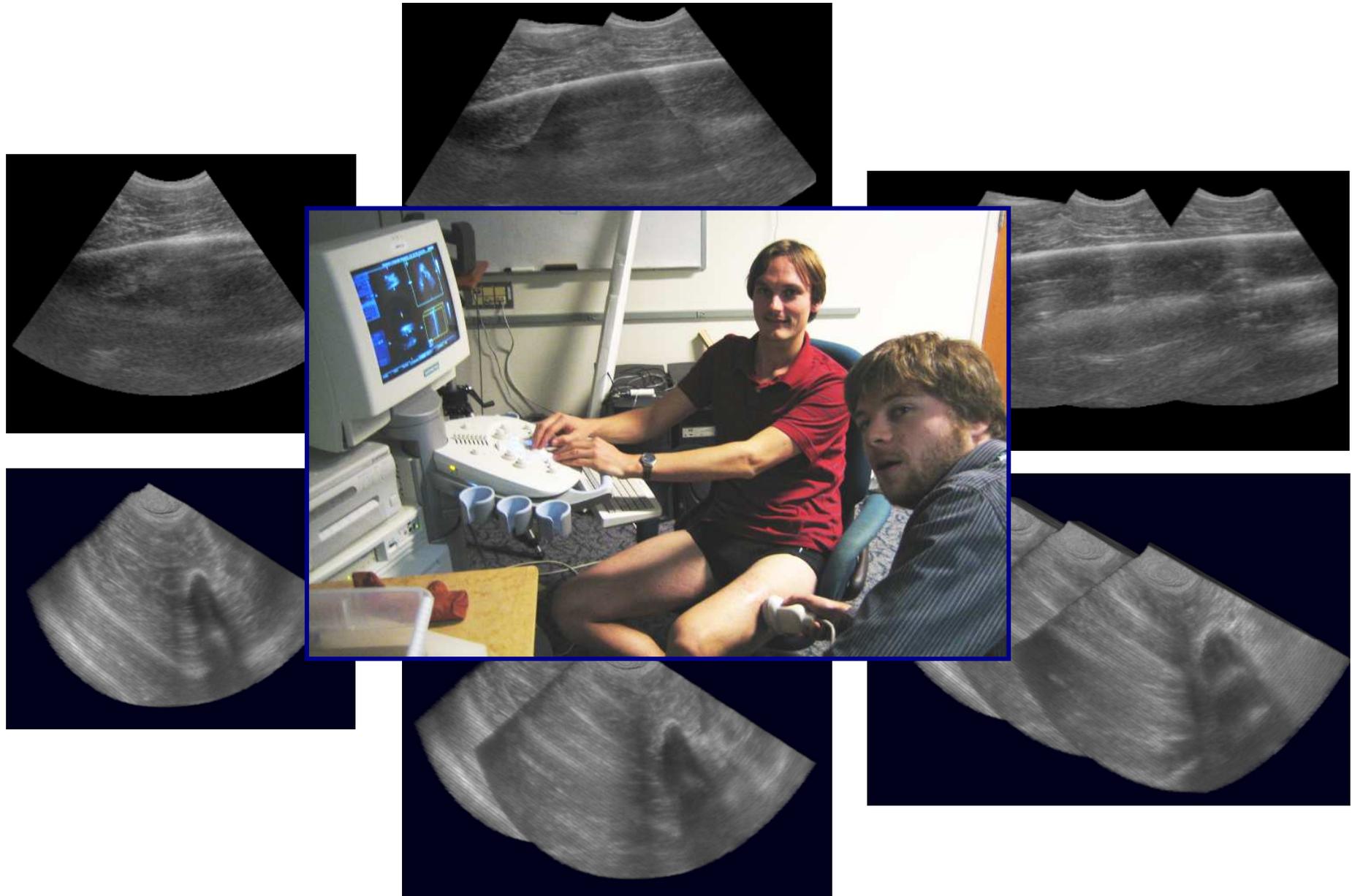


Surface decoration of small coloured components [...] closely set into an adhesive ground. Mosaic pieces are usually small squares, triangles, or other regular shapes.

Encyclopedia Britannica







# 3D Ultrasound Mosaicing

DA - final

Christian Wachinger

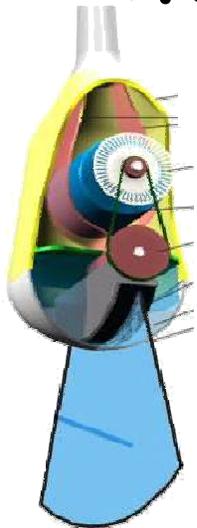
21 May 2007

Chair for Computer Aided Medical Procedures & Augmented Reality

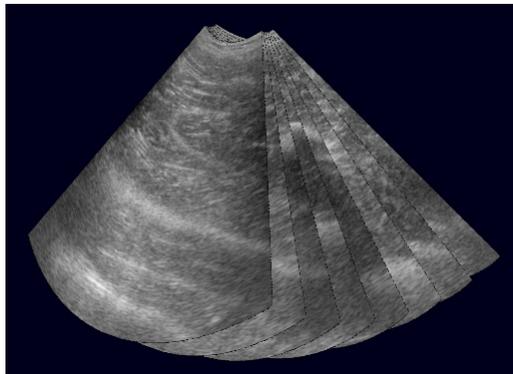
Department of Computer Science | Technische Universität München

# Moving from 2D to 3D imaging

1D Array

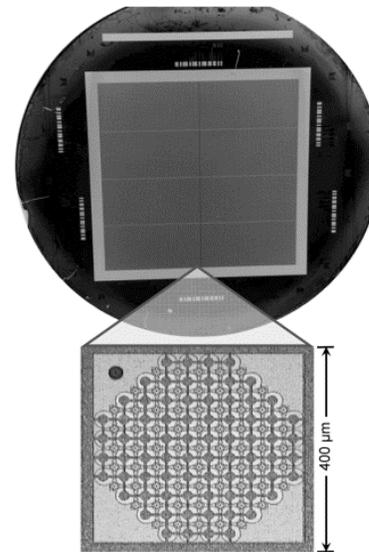


Wobbler



2D Array

CMUT Technology

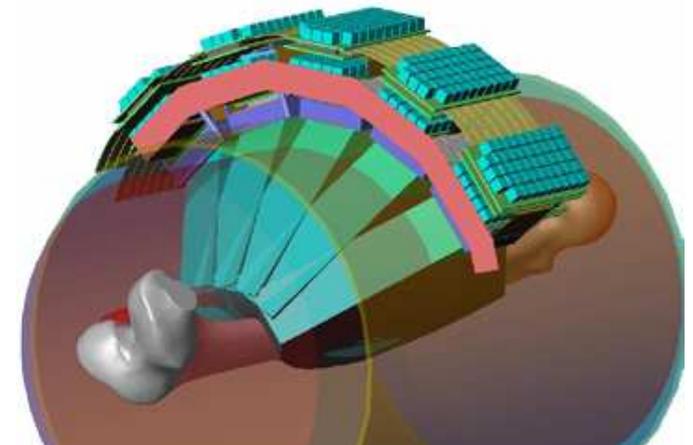


# Clinical value of Mosaicing

- Spatial relationship among structures that are too large for a single volume is easier to understand – [Kim et al., '03]
- Sonographers have the flexibility to visualize anatomical structures from a variety of different angles – [Peetrons et al., '02] – [Leung et al., '05]
- Size and distance measurements of large organs are possible – [Ying et al., '05]
- Individual structures within a broader context can be identified by having an image of the whole examination area – [Dietrich et al., '02]
- Specialists not used to ultrasound can better understand the spatial relationships of anatomical structures; helping to bridge the gap between the modalities and making it easier to convey sonographic findings to other experts – [Henrich et al., '03]

# Concrete applications on the short run

- Breast Scanner
  - Flexible plate between body and transducer to ensure uniform deformation
  - No radiation exposure
  - “Matrix Scanning”
  
- DARPA Project
  - Cuff around the arm or leg
  - Direct treatment of internal bleedings on the battlefield by non-experts
  - “Cylindric Scanning”



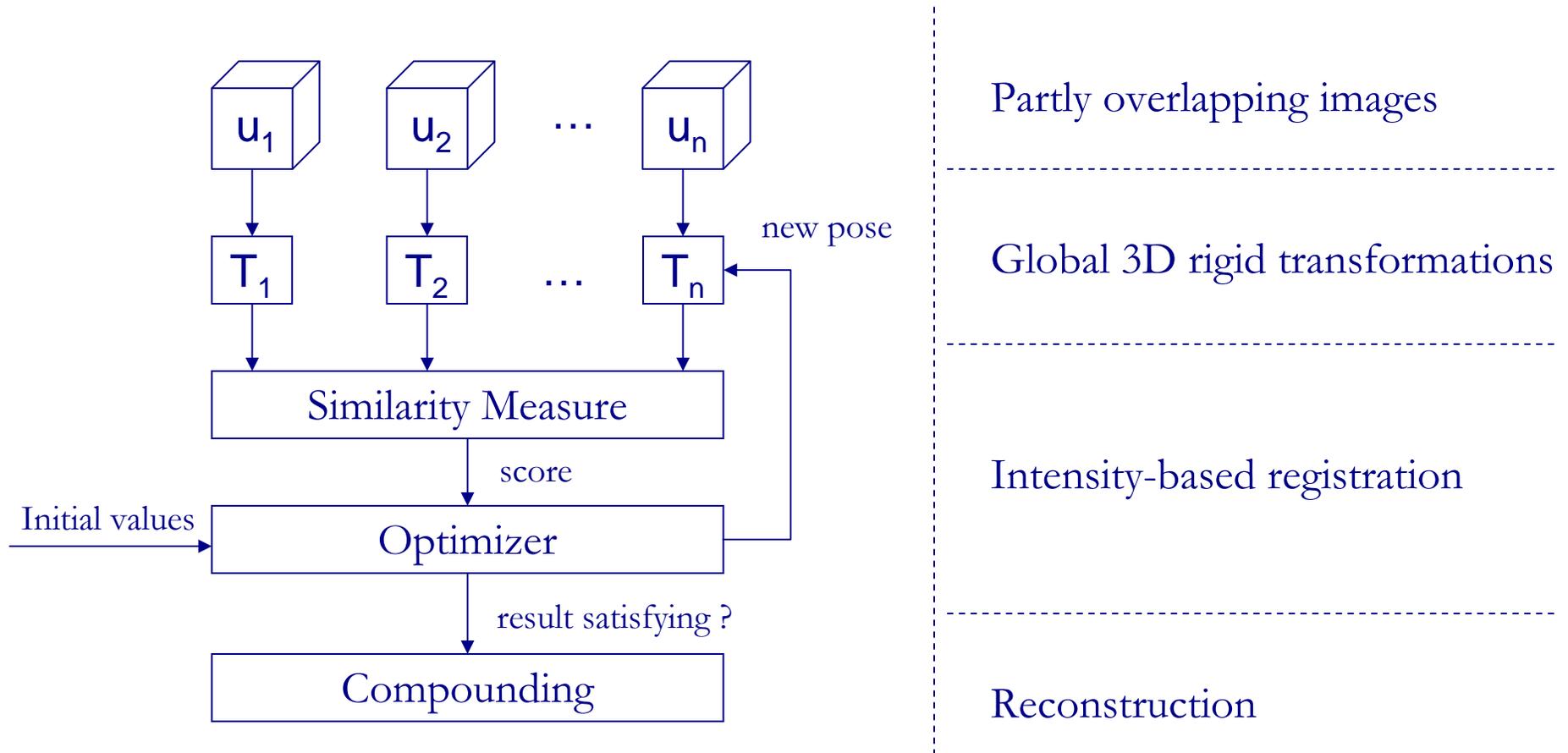
# Agenda

1. Registration Strategies
2. Similarity Measures
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

# Agenda

1. **Registration Strategies**
  1. Problem statement
  2. Pairwise Strategies
  3. Simultaneous Strategies
2. Similarity Measures
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

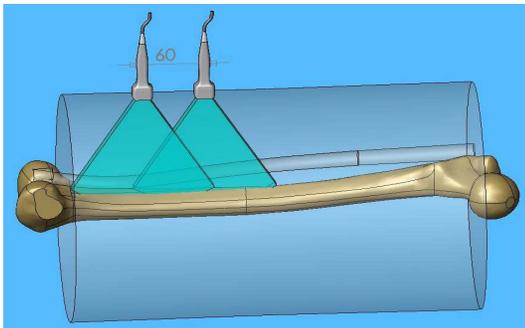
# Mosaicing Framework



# Problem statement

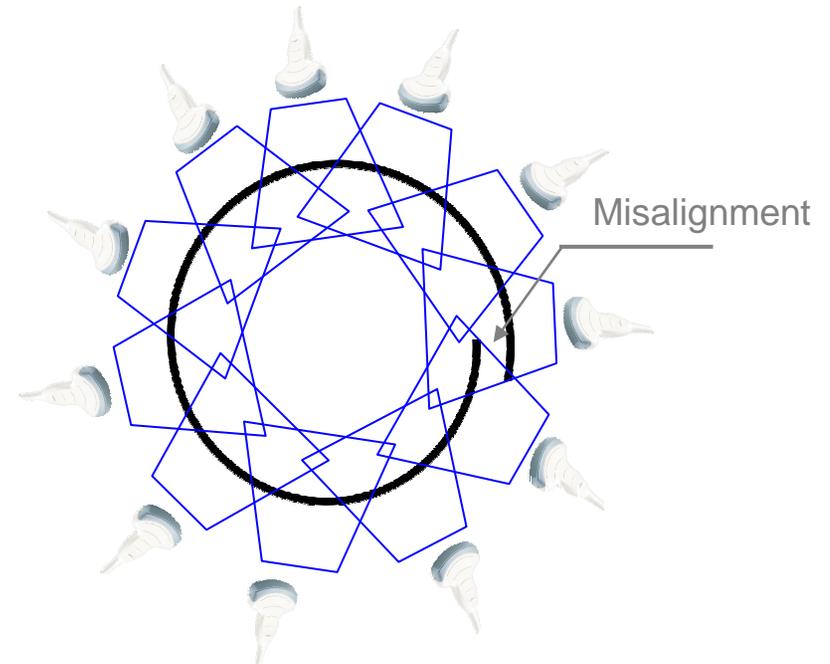
- In the literature proposed 3D mosaicing techniques by [Gee et al., '03] and [Poon et al., '06] use a **sequence of pairwise registrations**

## Partial Overlap:



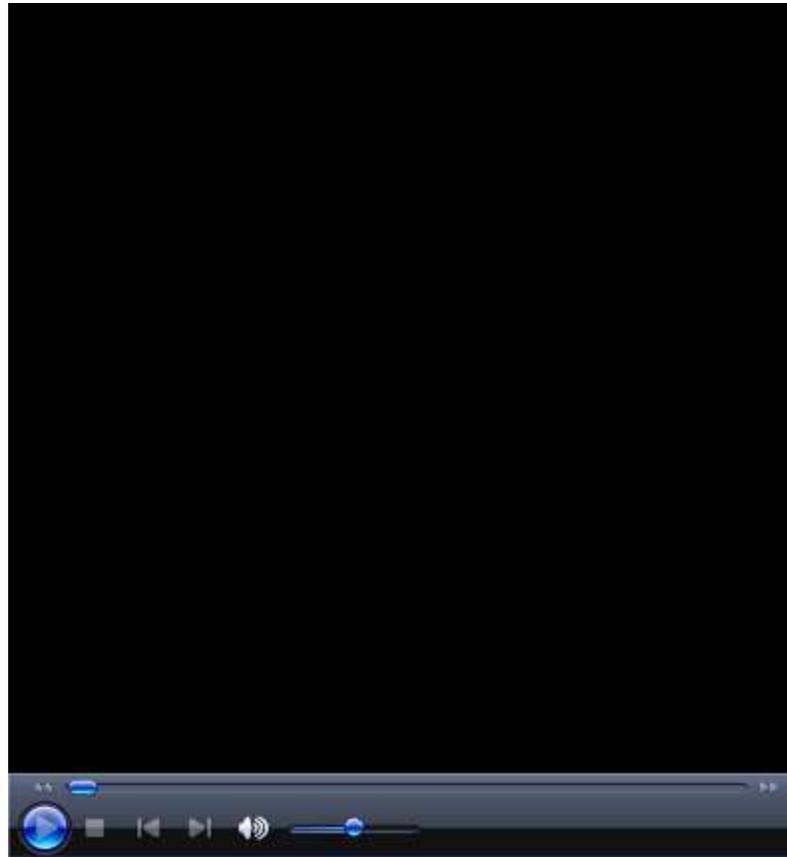
⇒ High demands on the overlap invariance of similarity measures

## Accumulated errors:



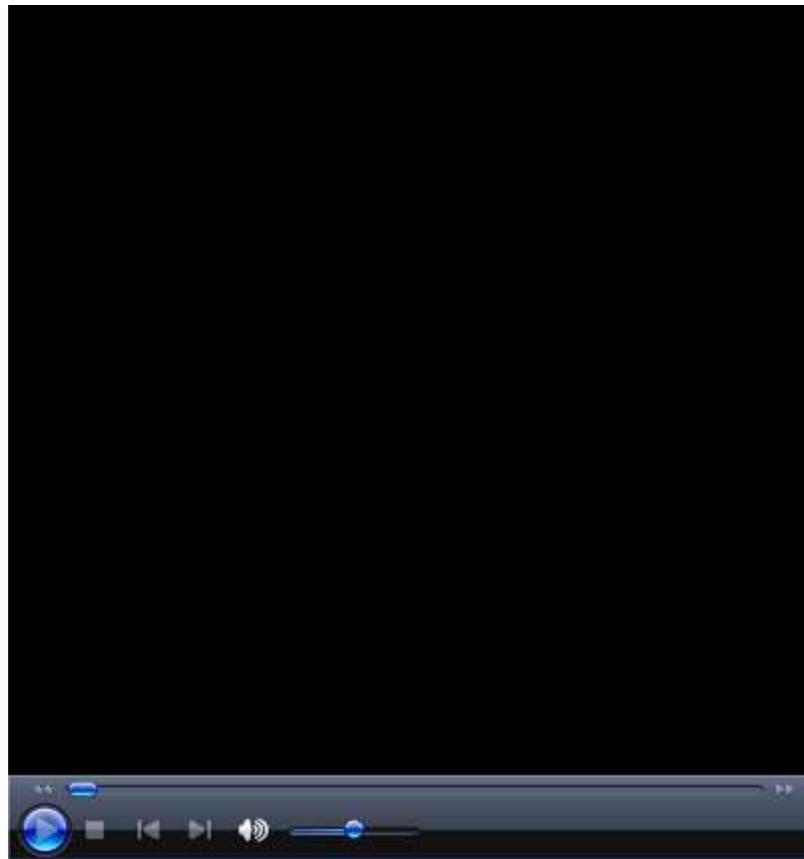
# Problem statement

- Partial overlap: ill-conditioned registration



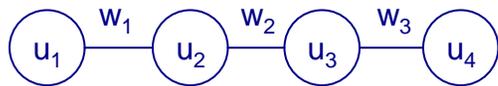
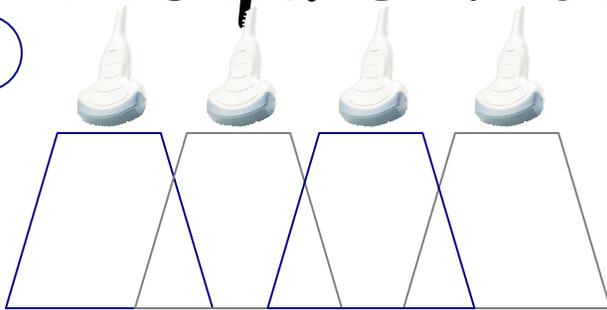
# Problem statement

- Accumulated Errors



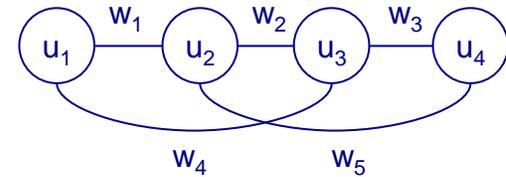
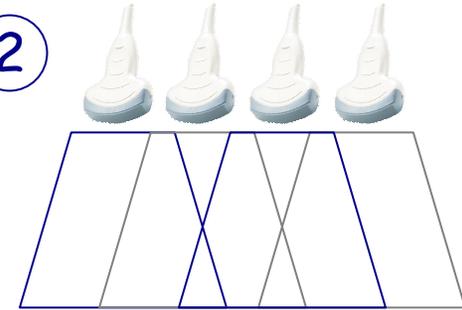
# Imaging Situations and Acquisition Graph

1



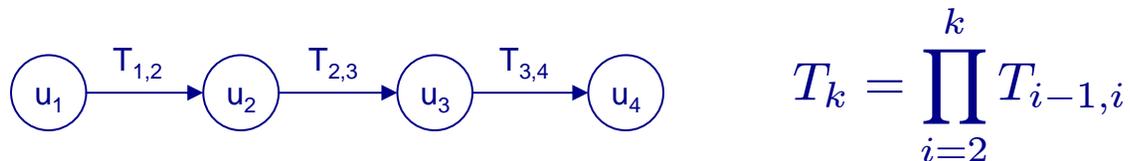
“Extended field-of-view”

2



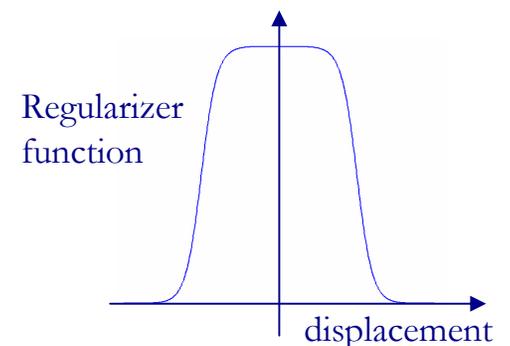
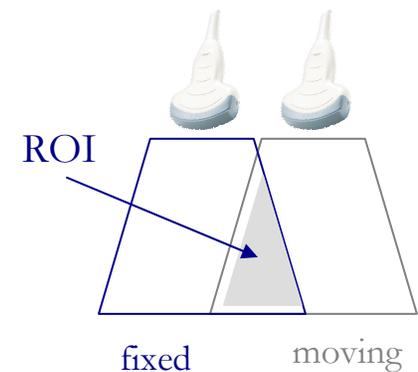
# Sequential Pairwise Registration

- Intensity-based registration with bivariate similarity measures



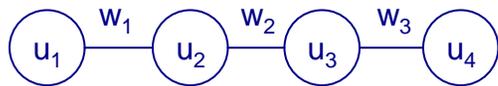
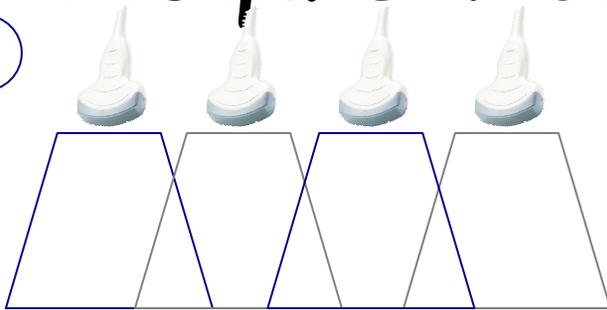
- Problem:** Overlap dependence of similarity measures

- Usage of an ROI that is a subset of the overlap  
 ⇒ But: Limited region of overlap further reduced
- Penalize large spatial displacements  
 ⇒ But: Manual adjustment to scenario



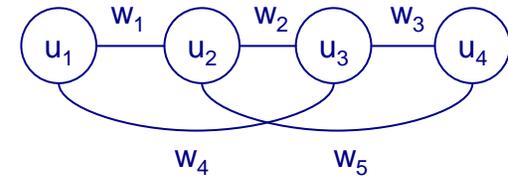
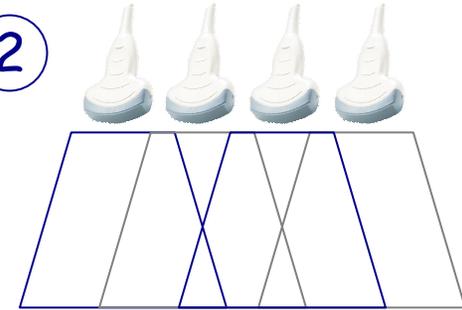
# Imaging Situations and Acquisition Graph

1

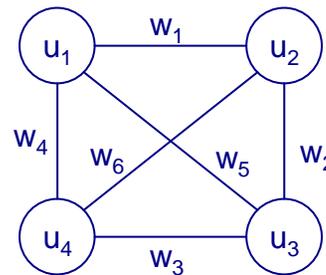
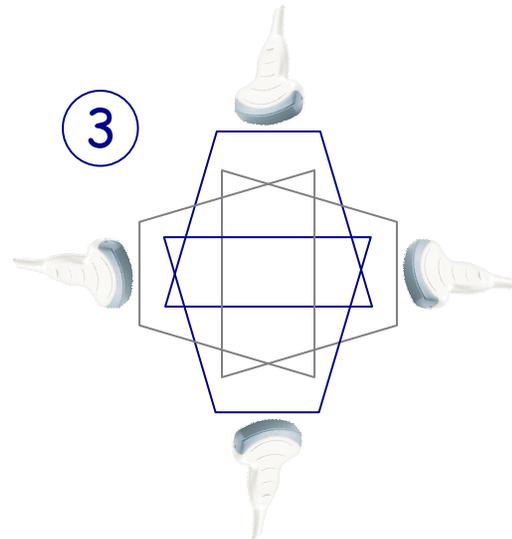


“Extended field-of-view”

2

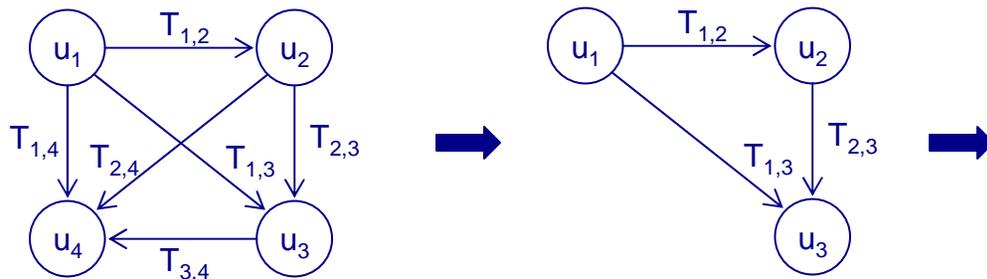


3



“Compounding”

# Complete Pairwise Registration



Theoretically:  $T_{1,3} = T_{1,2} * T_{2,3}$

Practically:  $T_{1,3} \approx T_{1,2} * T_{2,3}$

$\Rightarrow$  Transformation normalization

- Lie group normalization presented by Vercauteren et al. at MICCAI '06
- Bases on the work of Xavier Pennec:
  - Computing the mean of geometric features - Application to the mean rotation, '98
  - Uniform distribution, distance and expectation problems for geometric features processing, '98
  - Probabilities and Statistics on Riemannian Manifolds: Basic Tools for Geometric Measurements, '99
  - Intrinsic Statistics on Riemannian Manifolds: Basic Tools for Geometric Measurements, '06
  - Statistical Computing on Manifolds for Computational Anatomy, Habilitation, '06

# Estimation problems on Lie groups

- Distance between two points of a Lie group

$$\begin{aligned} \text{dist}(x, y) &= \text{dist}(\text{Id}, x^{-1} \circ y) \\ &= \|\log_{\text{Id}}(x^{-1} \circ y)\| \end{aligned}$$

- Exactly one Fréchet mean element exists

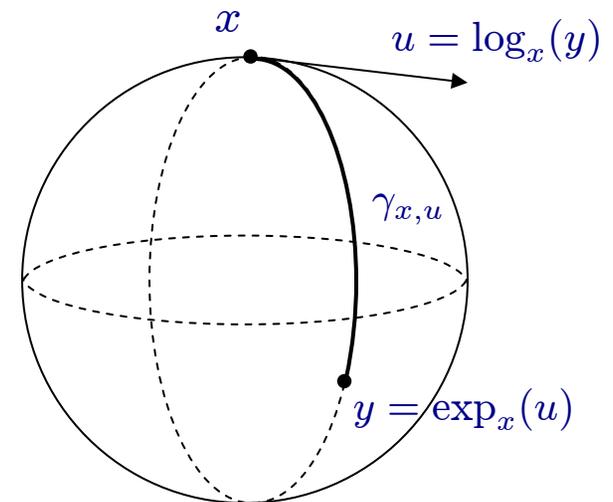
$$\mathcal{F}_x = \arg \min_{y \in \mathcal{G}} (\mathbb{E}[\text{dist}(y, x)^2])$$

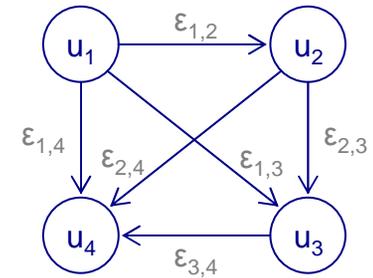
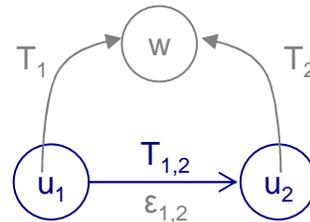
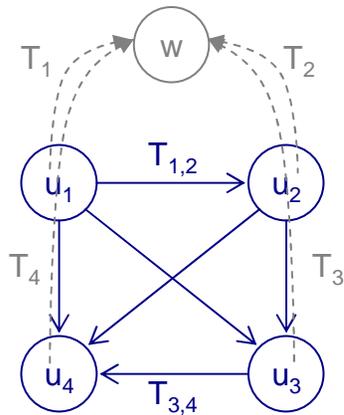
- The covariance matrix

$$\Sigma_{xx} = \mathbb{E} \left[ \log_{\mathbb{E}[x]}(x) \cdot \log_{\mathbb{E}[x]}(x)^T \right]$$

- Mahalanobis distance of a point  $y$  to a random feature  $x$

$$\mu_x^2(y) = \log_{\mathbb{E}[x]}(y)^T \cdot \Sigma_{xx}^{-1} \cdot \log_{\mathbb{E}[x]}(y)$$





$$\epsilon_{i,j} = T_i^{-1} \cdot T_j \cdot T_{i,j}$$

$$\min_{[T_1, \dots, T_n]} \sum_{(i,j)} \mu_{\epsilon}^2(\epsilon_{i,j})$$

Emphasizing pairs having high overlap with weights  $\omega_{i,j}$

$$[\hat{T}_1, \dots, \hat{T}_n] = \min_{[T_1, \dots, T_n]} \sum_{(i,j)} \omega_{i,j} \cdot \mu_{\epsilon}^2(\epsilon_{i,j})$$

# Registration with Lie group normalization

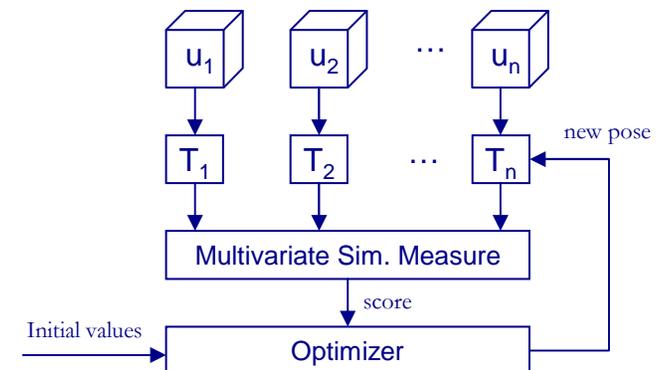
1. Start with initial global transformations  $\mathcal{T} = \{T_1, \dots, T_n\}$
2. Do
  - 2.1 Deduce initial pairwise transformations  $T_{i,j}$  from  $\mathcal{T}$
  - 2.2 Compute all pairwise registrations  $T_{i,j}$
  - 2.3 Estimate new  $\mathcal{T}$  from calculated  $T_{i,j}$  with Lie normalization
3. While ( $\varepsilon_t > \delta$ )
4. Return  $\mathcal{T}$

# Agenda

1. Registration Strategies
  1. Problem statement
  2. Registration Scenarios
  3. Pairwise Strategies
    1. Sequential
    2. Complete
  4. **Simultaneous Strategies**
    1. Full-Simultaneous
    2. Semi-Simultaneous
2. Similarity Measures
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

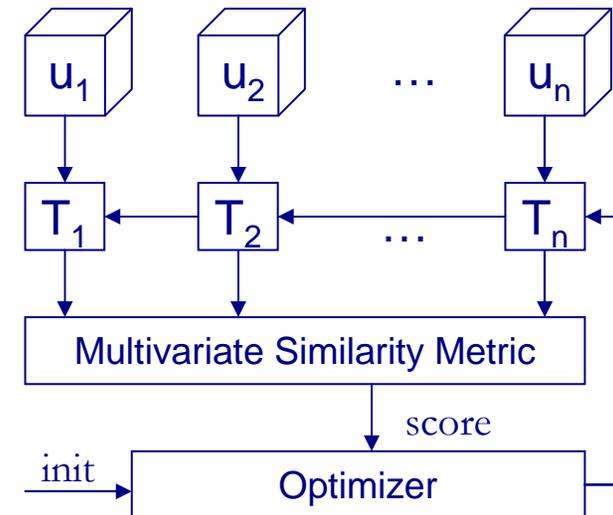
# Simultaneous registration

- Registration of all  $n$  images at the same time
  - Direct optimization of the global transformations –  $n*6$  parameters
  - Considering the whole imaging setup – **multivariate similarity metrics**
- Addressing the described problems
  1. Accumulation errors do not come up
  2. Better conditioned costfunction:
    - Overlap
    - Viewing angle dependence of US
- Increasing computational complexity
  - Higher dimensional parameter space
  - Evaluation of costfunction is more expensive



# The compromise: Semi-Simultaneous Registration

- Optimizing only one transformation at a time
- **But** using a multivariate similarity measure



Do

For each  $i$  in  $\{1, \dots, n\}$

Register  $u_i$  to  $\{u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n\}$

for num steps by optimizing  $T_i$

END

While alignment not satisfying

- Calculation of the Similarity Measure is faster
- Splitting up the high dimensional parameter space

# Registration Strategies - Overview

	Sequential PW	Complete PW	Semi-Simultaneous	Full-Simultaneous
Optimization	6	6	6	$n \cdot 6$
Similarity	Bivariate	Bivariate	Multivariate	Multivariate
Overlap	Sequence	All	All	All

# Agenda

1. Registration Strategies
2. Similarity Measures
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

# Agenda

1. Registration Strategies
2. **Similarity Measures**
  1. Bivariate Measures
  2. Multivariate Extensions
    1. Summed-up bivariate extension
    2. Joint density function
    3. Voxel-wise extension
  3. Practical issues
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

# Images as random variables

Pierre-Simon Laplace about probability theory:

*“It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge.”*

- Probability of one pixel to have the intensity  $\mathbf{i}$  in the image

- Image histogram has the role of the pdf

$$P(i) = \frac{N_i}{N}$$

- Maximum Likelihood Estimation (MLE) Framework

- Probability for the model  $\mathbf{m}$  having the observations  $\mathbf{a}$

$$P(a = \{a_1, \dots, a_n\} | m) = \prod_{i=1}^n P(a_i | m)$$

- Log likelihood function

$$\log \mathcal{L}(m | a) = \sum_{i=1}^n \log P(a_i | m)$$

# MLE framework for image alignment

- Imaging setup

$$u(x) = f(v(T(x))) + \varepsilon$$

$u, v$  : images

$\varepsilon$  : Gaussian noise

$f$  : intensity mapping

- Probability function

$$P(u|v, T, \varepsilon, f) = P(\varepsilon = u(x) - f(v(T(x_k))))$$

- Negative log-likelihood function

$$\begin{aligned} -\log \mathcal{L}(T, \varepsilon, f) &= -\log P(u|v, T, \varepsilon, f) \\ &= -\log P(\varepsilon = u(x) - f(v(T(x_k)))) \end{aligned}$$

- Stationary white Gaussian noise

$$P(u|v, T, f, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2} \frac{(u - f(v^\downarrow))^2}{\sigma^2}}$$

$$v^\downarrow = v(T(.))$$

# MLE framework for image alignment

- Negative log-likelihood function

$$P(u|v, T, f, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(u - f(v^\downarrow))^2}{\sigma^2}}$$

$$\begin{aligned} -\log \mathcal{L}(T, f, \sigma) &= - \sum_{x_k \in \Omega_u} \log P(u(x_k)|v^\downarrow(x_k), f, \sigma) \\ &= N \log \sqrt{2\pi}\sigma + \frac{1}{2} \sum_{x_k \in \Omega_u} \frac{(u(x_k) - f(v^\downarrow(x_k)))^2}{\sigma^2} \end{aligned}$$

- Assuming the identity as intensity map

$$\begin{aligned} -\log \mathcal{L}(T, f, \sigma) &= N \log \sqrt{2\pi}\sigma + \frac{1}{2} \sum_{x_k \in \Omega_u} \frac{(u(x_k) - v^\downarrow(x_k))^2}{\sigma^2} \\ &\propto \mathbb{E} [(u(x_k) - v^\downarrow(x_k))^2] \end{aligned}$$

➔ Sum of Squared Differences

# MLE framework for image alignment

- Intensity mapping that allows for an affine transformation  $u \approx \alpha \cdot v + \beta$

⇒ using normalized images

$$\tilde{u}(x) = \frac{u(x) - \mathbb{E}[u(X)]}{\text{Var}(u(X))}$$

- Normalized Sum of Squared Differences

$$\begin{aligned} \text{NSSD}(T) &= \mathbb{E}[(\tilde{v}(T(X)) - \tilde{u}(X))^2] \\ &= \mathbb{E}[\tilde{v}^2(T(X)) - 2\tilde{v}(T(X))\tilde{u}(X) + \tilde{u}^2(X)] \\ &= \mathbb{E}[\tilde{v}^2(T(X))] - 2\mathbb{E}[\tilde{v}(T(X))\tilde{u}(X)] + \mathbb{E}[\tilde{u}^2(X)] \\ &= 1 - 2 \frac{\mathbb{E}[v(T(X))u(X)] - \mathbb{E}[v(T(X))]\mathbb{E}[u(X)]}{\text{Var}(v(T(X))) \text{Var}(u(X))} + 1 \\ &= 2 - 2 \cdot \text{NCC}(v(T(X)), u(X)) \end{aligned}$$

➡ Normalized Cross Correlation

# MLE framework for image alignment

- By further varying the assumptions for the intensity mapping  $f$ , Correlation Ratio (CR) and Mutual Information (MI) can be deduced
  - Paul A. Viola, “Alignment by Maximization of Mutual Information”, ‘95
  - Roche et al., “Unifying Maximum Likelihood Approaches in Medical Image Registration”, ‘00
  
- Identity relationship: Sum of Squared Differences, Cross-Correlation
- Affine relationship: Normalized Cross Correlation
- Functional relationship: Correlation Ratio
- Statistical relationship: Mutual Information

# Extension of MLE to multiple images

## 1. Summed-up bivariate extension

$u_1, \dots, u_n$  : images  
 $\varepsilon_1, \dots, \varepsilon_n$  : Gaussian noises  
 $f_1, \dots, f_n$  : intensity mappings

$$\begin{aligned}
 -\mathcal{L}(T) &= -P(u_1^\downarrow | u_2^\downarrow, \dots, u_n^\downarrow, \vec{f}, \vec{\varepsilon}) \\
 &= -P(\varepsilon_2 = u_1^\downarrow - f_2(u_2^\downarrow), \varepsilon_3 = u_1^\downarrow - f_3(u_3^\downarrow), \dots, \varepsilon_n = u_1^\downarrow - f_n(u_n^\downarrow))
 \end{aligned}$$

$$= -\prod_{i=2}^n P(\varepsilon_i = u_1^\downarrow - f_i(u_i^\downarrow))$$

$$= -\prod_{i=2}^n \prod_{x_k \in \Omega} P(\varepsilon_i = u_1^\downarrow(x_k) - f_i(u_i^\downarrow(x_k)))$$

$$-\log(\mathcal{L}(T)) = -\sum_{i=2}^n \sum_{x_k \in \Omega} \log P(\varepsilon_i = u_1^\downarrow(x_k) - f_i(u_i^\downarrow(x_k)))$$

$$= -\sum_{i=2}^n \sum_{x_k \in \Omega} \log P(u_1^\downarrow | u_i^\downarrow, f_i, \varepsilon_i)$$

Bivariate formula

# Extension of MLE to multiple images

- Multivariate extensions by summing up the bivariate terms  $\sum_{i=2}^n SM(u_1, u_i)$
- Corresponds to summing up the first line of the similarity matrix M

$$M^{\nabla} = \begin{pmatrix} SM(u_1, u_1) & SM(u_1, u_2) & SM(u_1, u_3) & \dots & SM(u_1, u_n) \\ SM(u_2, u_1) & SM(u_2, u_2) & SM(u_2, u_3) & \dots & SM(u_2, u_n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ SM(u_n, u_1) & SM(u_n, u_2) & \dots & SM(u_n, u_{n-1}) & SM(u_n, u_n) \end{pmatrix}$$

- Focus on one image  $\Rightarrow$  suits well for semi-simultaneous registration
- Full-simultaneous variant by summing up the matrix

$$\sum_{i < j}^n SM(u_i, u_j)$$

# Interim summary - Multivariate Measures

	Pairwise	Semi-Simultaneous	Full-Simultaneous
SSD	$\mathbb{E}[(u - v^\downarrow)^2]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[(u_1 - u_i^\downarrow)^2]$	$\sum_{i<j} \omega_{i,j} \mathbb{E}[(u_i^\downarrow - u_j^\downarrow)^2]$
NCC	$\mathbb{E}[\tilde{u} \cdot \tilde{v}^\downarrow]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[\tilde{u}_1 \cdot \tilde{u}_i^\downarrow]$	$\sum_{i<j} \omega_{i,j} \mathbb{E}[\tilde{u}_i^\downarrow \cdot \tilde{u}_j^\downarrow]$
CR	$\frac{\text{Var}[\mathbb{E}(u v^\downarrow)]}{\text{Var}(u)}$	$\sum_{i=2}^n \omega_{1,i} \frac{\text{Var}[\mathbb{E}(u_1 u_i^\downarrow)]}{\text{Var}(u_1)}$	$\sum_{i<j} \omega_{i,j} \frac{\text{Var}[\mathbb{E}(u_i^\downarrow u_j^\downarrow)]}{\text{Var}(u_i^\downarrow)}$
MI	$\text{MI}(u, v^\downarrow)$	$\sum_{i=2}^n \omega_{1,i} \text{MI}(u_1, u_i^\downarrow)$	$\sum_{i<j} \omega_{i,j} \text{MI}(u_i^\downarrow, u_j^\downarrow)$

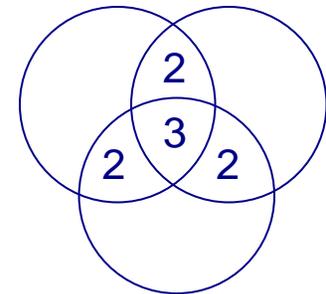
# Extension of MLE to multiple images

## 2. Joint density function

$$\begin{aligned} -\log(\mathcal{L}(T)) &= -\log P(u_1^\downarrow, u_2^\downarrow, \dots, u_n^\downarrow) \\ &= -\sum_{x_k \in \Omega} \log P(u_1^\downarrow(x_k), u_2^\downarrow(x_k), \dots, u_n^\downarrow(x_k)) \end{aligned}$$

- Curse of dimensionality: Observations grow linearly, needed data for reliable estimation grows exponentially
- Not flexible to varying numbers of overlapping images

⇒ Not used for Mosaicing

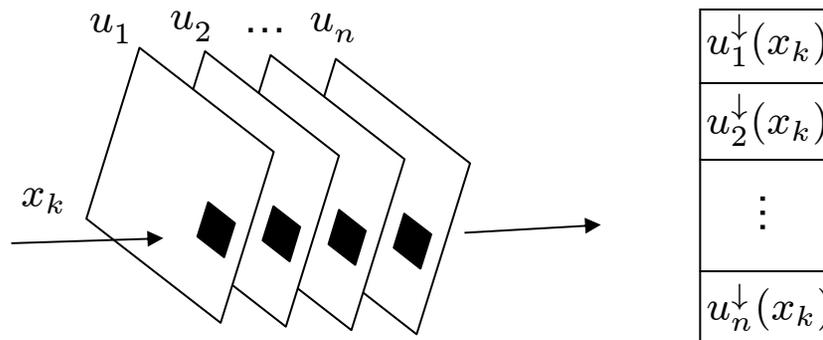


# Extension of MLE to multiple images

## 3. Voxel-wise extension

$$\begin{aligned}
 -\log(\mathcal{L}(T)) &= -\log P(u_1^\downarrow, u_2^\downarrow, \dots, u_n^\downarrow) \\
 &= -\sum_{x_k \in \Omega} \log P^k(u_1^\downarrow(x_k), u_2^\downarrow(x_k), \dots, u_n^\downarrow(x_k))
 \end{aligned}$$

- Independent but not identical distributed coordinate samples
- Allows for varying numbers of overlapping images
- Idea of pixel stack was introduced by Miller '02



# Voxel-wise extension for SSD

$f_1, \dots, f_n$  : intensity mappings

$$\begin{aligned}
 -\log(\mathcal{L}(T)) &= -\frac{1}{N} \sum_{x_k \in \Omega} \log P^k(u_1^\downarrow(x_k), \dots, u_n^\downarrow(x_k)) \\
 &\approx -\frac{1}{N} \sum_{x_k \in \Omega} \log \prod_{i=1}^n P^k(u_i^\downarrow(x_k)) \\
 &= -\frac{1}{N} \sum_{x_k \in \Omega} \sum_{i=1}^n \log \left( \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(f_i(u_i^\downarrow(x_k)) - \mu_k)^2}{\sigma_k^2}} \right) \\
 &= -\frac{1}{N} \sum_{x_k \in \Omega} \sum_{i=1}^n \left( \log \frac{1}{\sqrt{2\pi}\sigma} - \frac{1}{2} \frac{(f_i(u_i^\downarrow(x_k)) - \mu_k)^2}{\sigma_k^2} \right) \\
 &\approx \frac{1}{N} \sum_{x_k \in \Omega} \frac{1}{\sigma_k^2} \sum_{i=1}^n (f_i(u_i^\downarrow(x_k)) - \mu_k)^2
 \end{aligned}$$

# Voxel-wise extension for SSD

- Assuming identity as intensity mapping

$$-\log(\mathcal{L}(T)) \approx \frac{1}{N} \sum_{x_k \in \Omega} \frac{1}{\sigma_k^2} \sum_{i=1}^n (u_i^\downarrow(x_k) - \mu_k)^2$$

- Sum of voxel-wise variances (SVV)**

$$\begin{aligned} \text{SVV}(u_1, \dots, u_n) &= \frac{1}{N} \sum_{x_k \in \Omega} \frac{1}{m_k} \sum_{i=1}^{m_k} (u_{\iota(i)}^\downarrow(x_k) - \mu_k)^2 \\ &\approx \sum_{x_k \in \Omega} \omega_k \cdot \mathbb{E}_i \left[ (u_{\iota(i)}^\downarrow(x_k) - \mu_k)^2 \right] \end{aligned}$$

$$\mu_k = \frac{1}{m_k} \sum_{i=1}^{m_k} u_{\iota(i)}^\downarrow(x_k).$$

$m_k$  : overlapping images at  $x_k$   
 $\iota : \{1, \dots, m_k\} \rightarrow \{1, \dots, n\}$   
 $\omega_k$  : weighting coefficient

# Voxel-wise extension for MI

- **Congealing** [Lilla Zöllei '06, Miller '02]

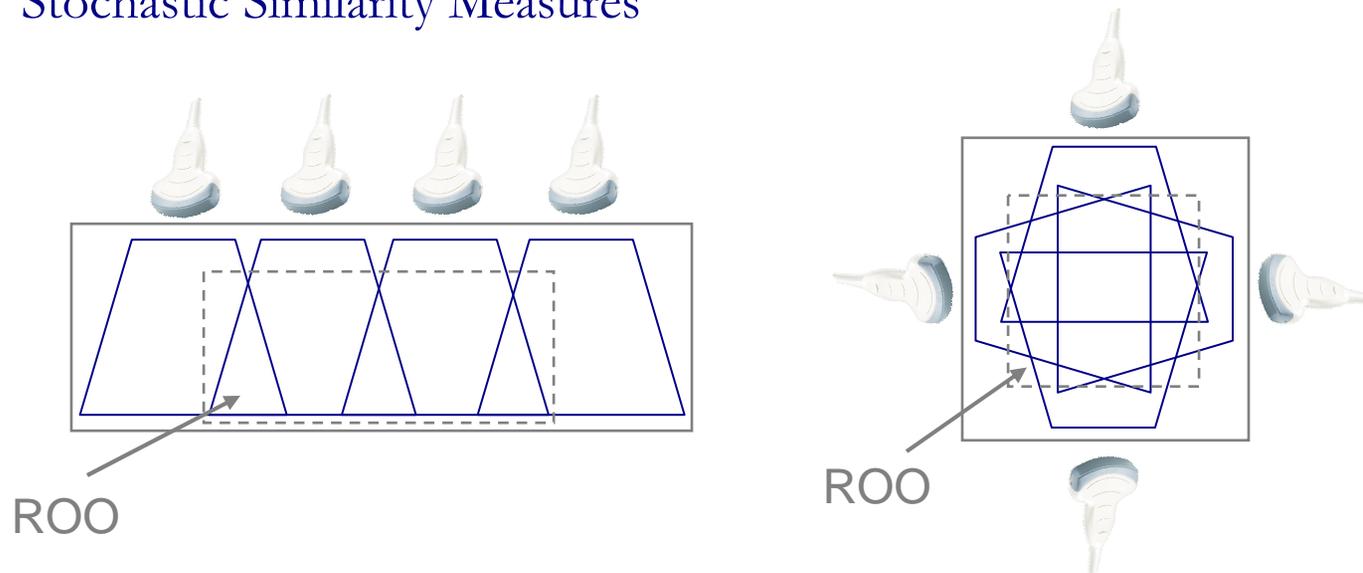
$$\begin{aligned}\log(\mathcal{L}(T)) &= \frac{1}{N} \sum_{x_k \in \Omega} \log P^k(u_1^\downarrow(x_k), \dots, u_n^\downarrow(x_k)) \\ &\approx \frac{1}{N} \sum_{x_k \in \Omega} \log \prod_{i=1}^n P^k(u_i^\downarrow(x_k)) \\ &= \frac{1}{N} \sum_{x_k \in \Omega} \sum_{i=1}^n \log P^k(u_i^\downarrow(x_k)) \\ &\approx \sum_{x_k \in \Omega} \omega_k \cdot H(P^k)\end{aligned}$$

# Summary - Multivariate Similarity Measures

	Pairwise	Semi-Simultaneous	Full-Simultaneous	Voxel-Wise
SSD	$\mathbb{E}[(u - v^\downarrow)^2]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[(u_1 - u_i^\downarrow)^2]$	$\sum_{i<j} \omega_{i,j} \mathbb{E}[(u_i^\downarrow - u_j^\downarrow)^2]$	$\sum_{x_k \in \Omega} \omega_k \mathbb{E}_i[(\mu_k - u_i^\downarrow(x_k))^2]$
NCC	$\mathbb{E}[\tilde{u} \cdot \tilde{v}^\downarrow]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[\tilde{u}_1 \cdot \tilde{u}_i^\downarrow]$	$\sum_{i<j} \omega_{i,j} \mathbb{E}[\tilde{u}_i^\downarrow \cdot \tilde{u}_j^\downarrow]$	$\sum_{x_k \in \Omega} (\omega_k \cdot \tilde{u}_1^\downarrow \cdot \tilde{u}_2^\downarrow \cdots \tilde{u}_n^\downarrow)$
CR	$\frac{\text{Var}[\mathbb{E}(u v^\downarrow)]}{\text{Var}(u)}$	$\sum_{i=2}^n \omega_{1,i} \frac{\text{Var}[\mathbb{E}(u_1 u_i^\downarrow)]}{\text{Var}(u_1)}$	$\sum_{i<j} \omega_{i,j} \frac{\text{Var}[\mathbb{E}(u_i^\downarrow u_j^\downarrow)]}{\text{Var}(u_i^\downarrow)}$	-
MI	$\text{MI}(u, v^\downarrow)$	$\sum_{i=2}^n \omega_{1,i} \text{MI}(u_1, u_i^\downarrow)$	$\sum_{i<j} \omega_{i,j} \text{MI}(u_i^\downarrow, u_j^\downarrow)$	$\sum_{x_k \in \Omega} \omega_k \text{H}(P^k)$

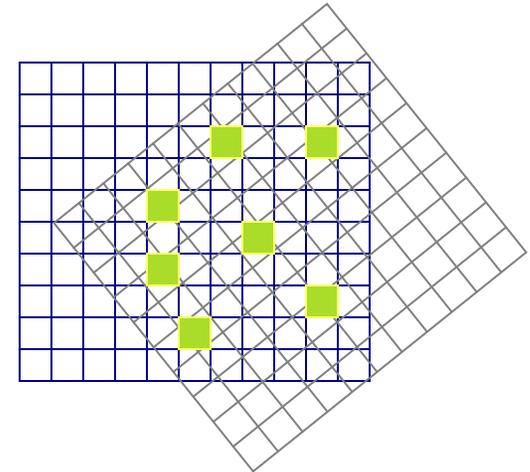
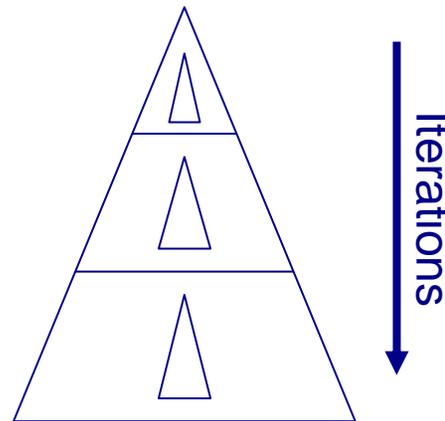
# Practical Issues

- Normalization of transformations → no sliding
- Speeding up the process
  - Region of overlap (ROO)
  - Semi-simultaneous approach with partial compounding
  - Stochastic Similarity Measures



# Stochastic Similarity Measures

- Taking samples of the similarity of images only at random locations
- [Zöllei '06] used 0.05% - 2.5%
- Enormous speed-up
- Fine-grained **stochastic pyramids** good complement to coarse-grained Gaussian pyramid



# Agenda

1. Registration Strategies
2. Similarity Measures
3. Reconstruction
4. Experiments & Results
5. Conclusion & Further Development

# General compounding methods

1. Selection:  $\text{Sel}_k^j = x_k^j$

$$X_k = \{x_k^1, \dots, x_k^m\}$$

2. Mean:  $\text{Mean}_k = \mathbb{E}[X_k]$

3. Median:  $\text{Median}_k = \text{median}\{x_k^1, \dots, x_k^m\}$

4. Max:  $\text{Max}_k = \max\{x_k^1, \dots, x_k^m\}$

5. Min:  $\text{Min}_k = \min\{x_k^1, \dots, x_k^m\}$

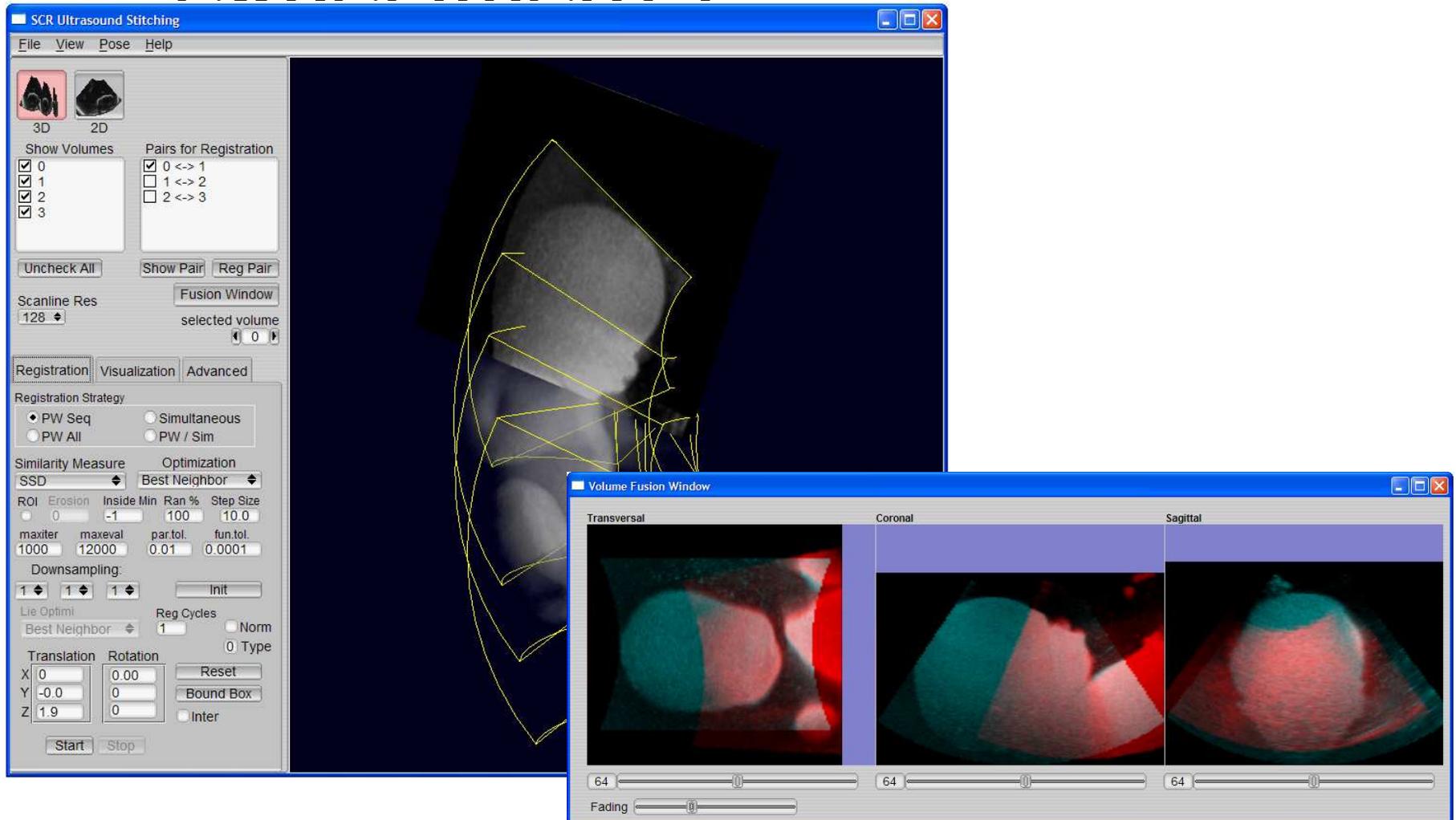
6. Geometric Mean:

$$\text{Geometric Mean}_k = \sqrt[n]{x_k^1 \cdot x_k^2 \cdot \dots \cdot x_k^m}$$

# Agenda

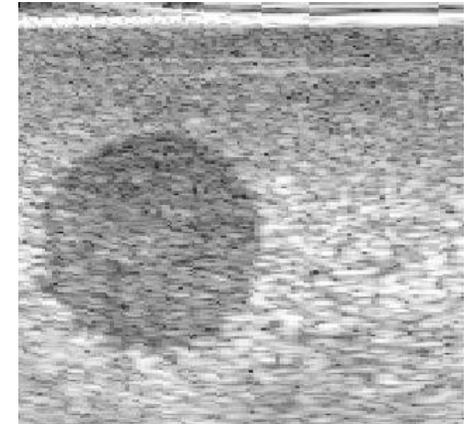
1. Registration Strategies
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# Mosaicing prototype for experiments



# US specific image pre-processing

- Speckle noise reduction
  - Low-pass filtering
- Shadow suppression
  - Shadow has negative influence on registration and compounding



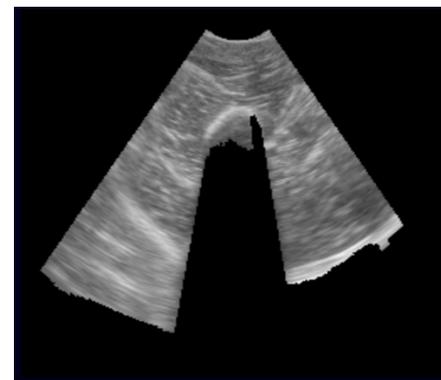
Original



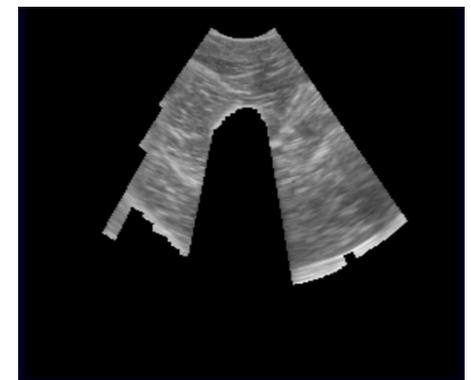
Intensity



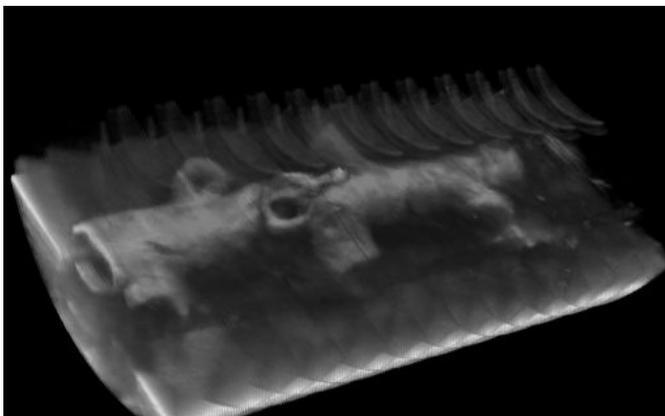
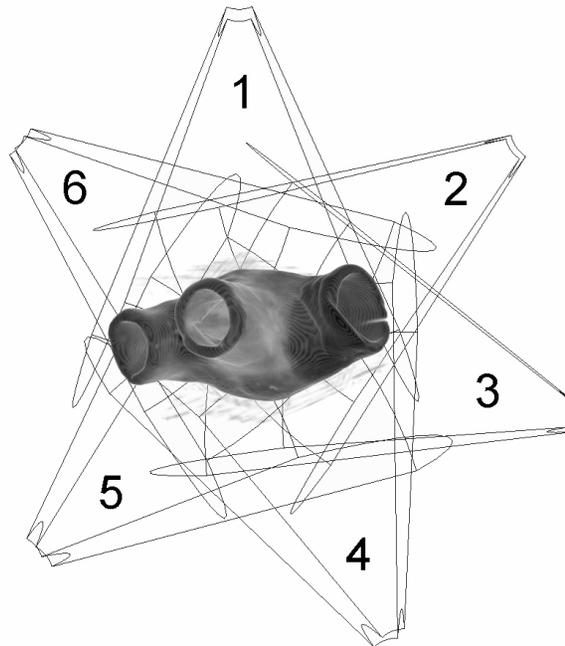
Variance



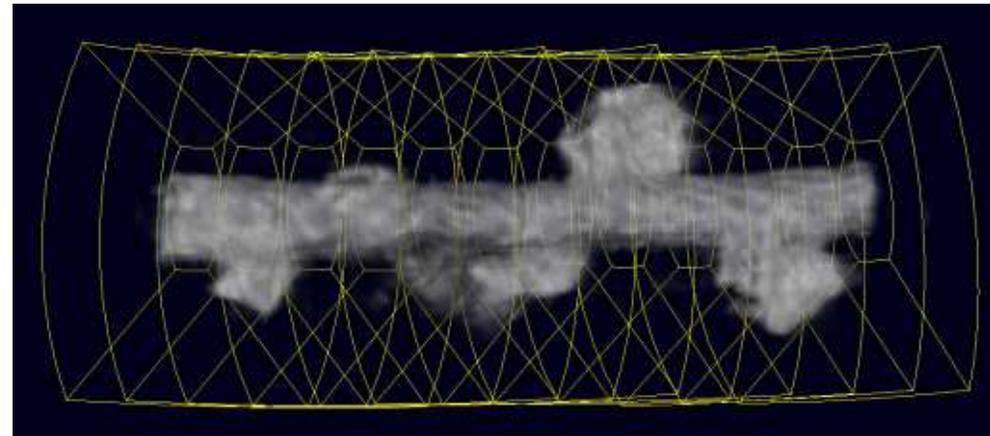
Maximum



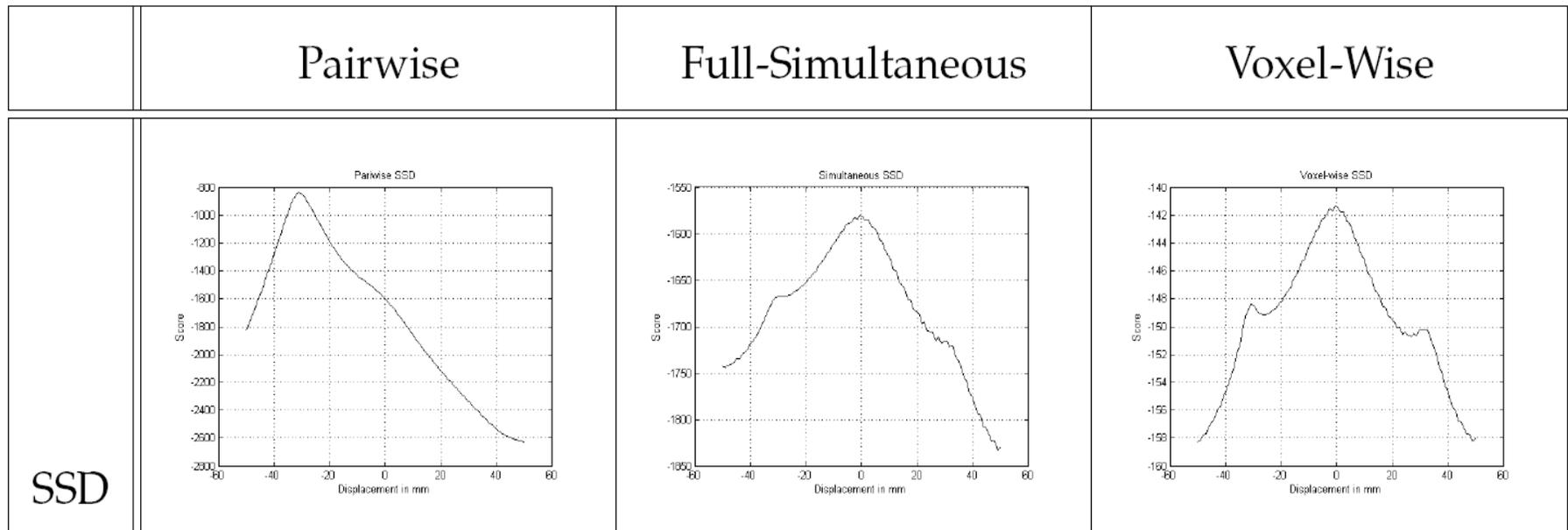
# Clay models



# Clay spine

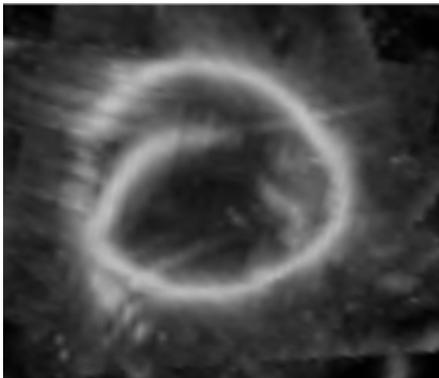


Similarity plot:

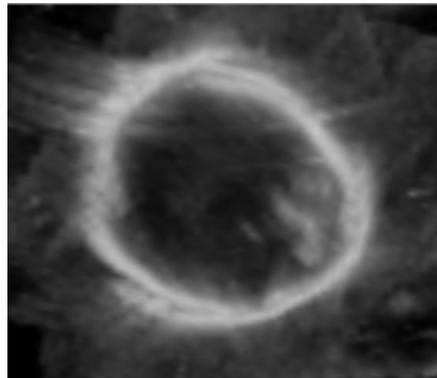


# Clay vessel

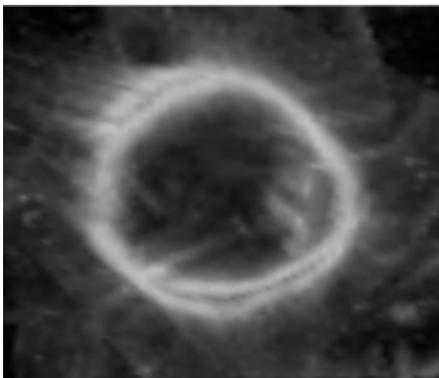
Pairwise



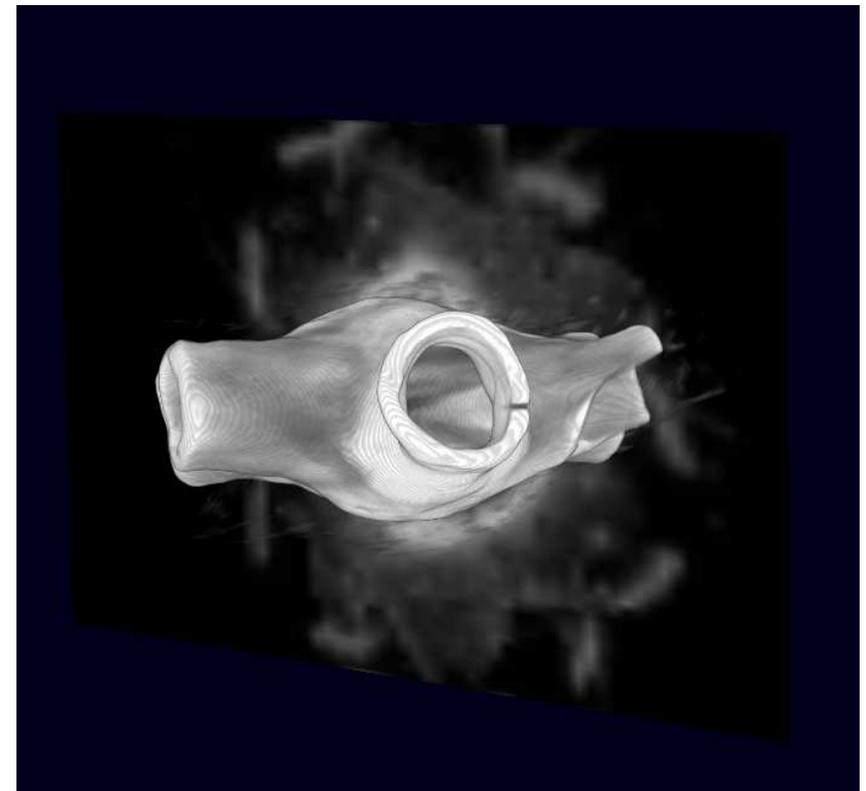
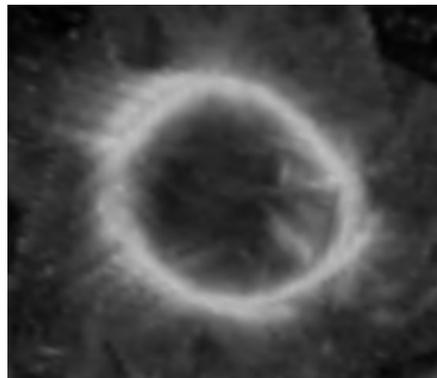
Lie normalization



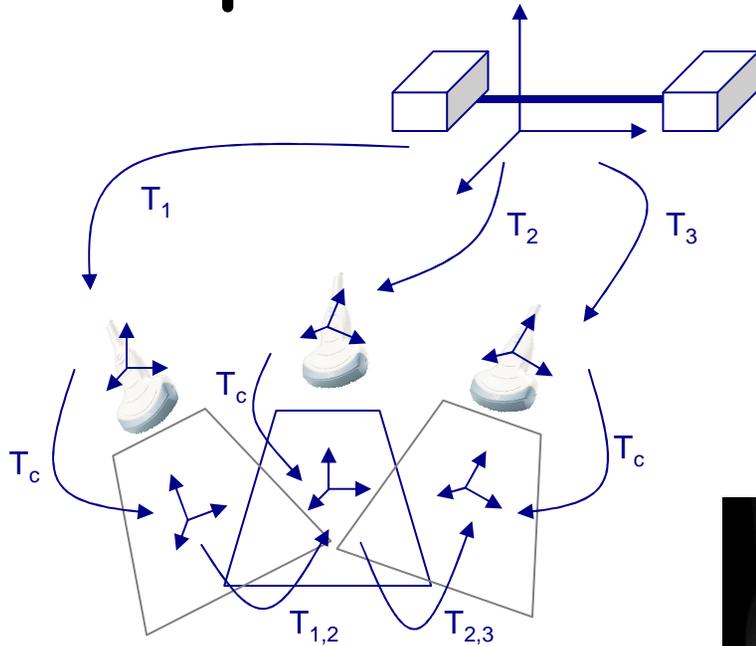
Semi-Simultaneous



Full-Simultaneous

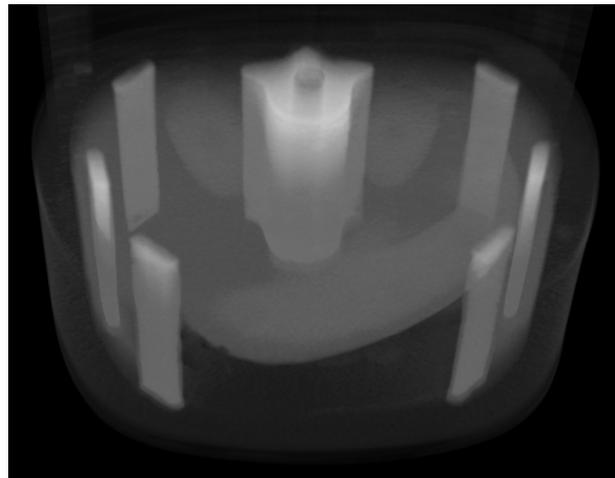


# Abdominal phantom - with optical tracking

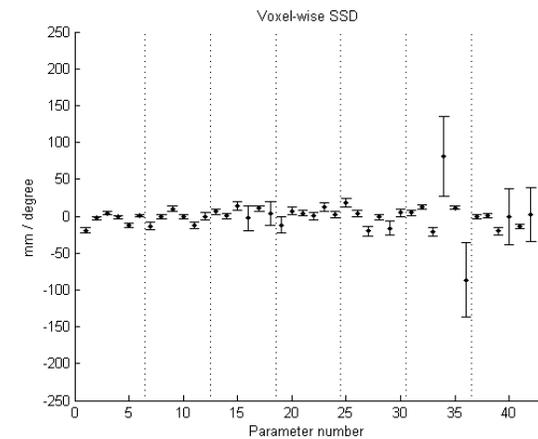
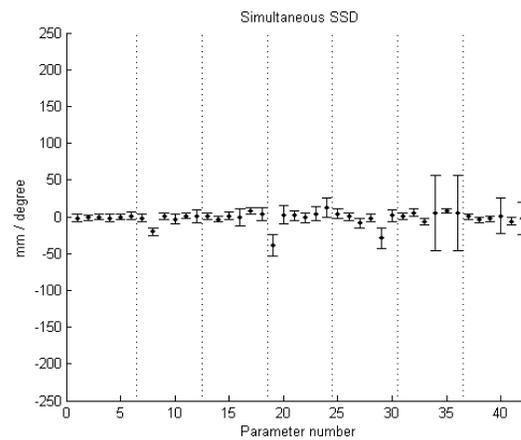
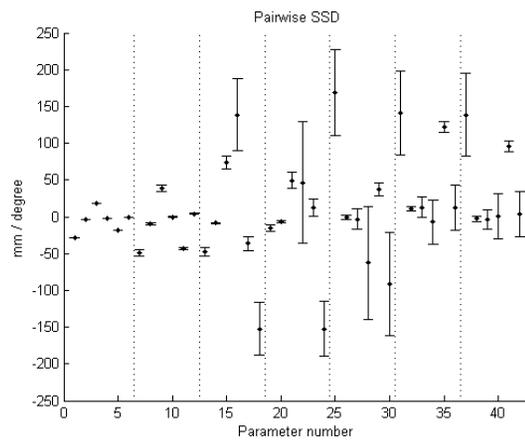


$$T_{1,2} T_c = T_c T_2 T_1^{-1}$$

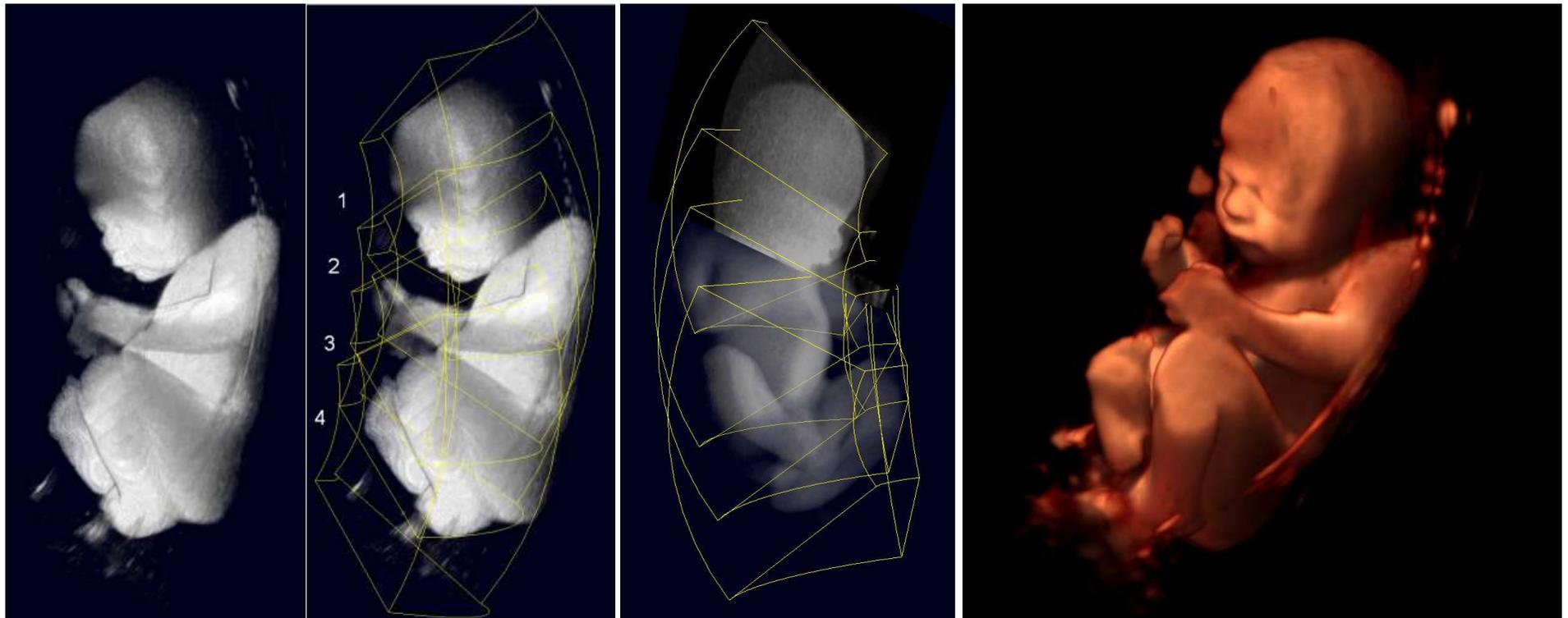
$$A \cdot X = X \cdot B$$

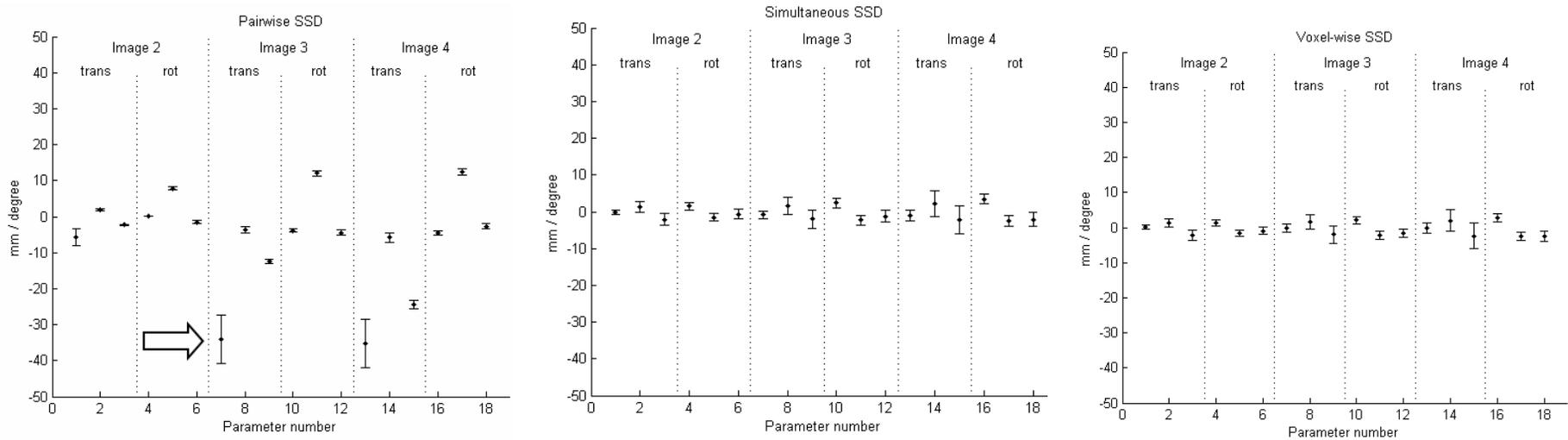
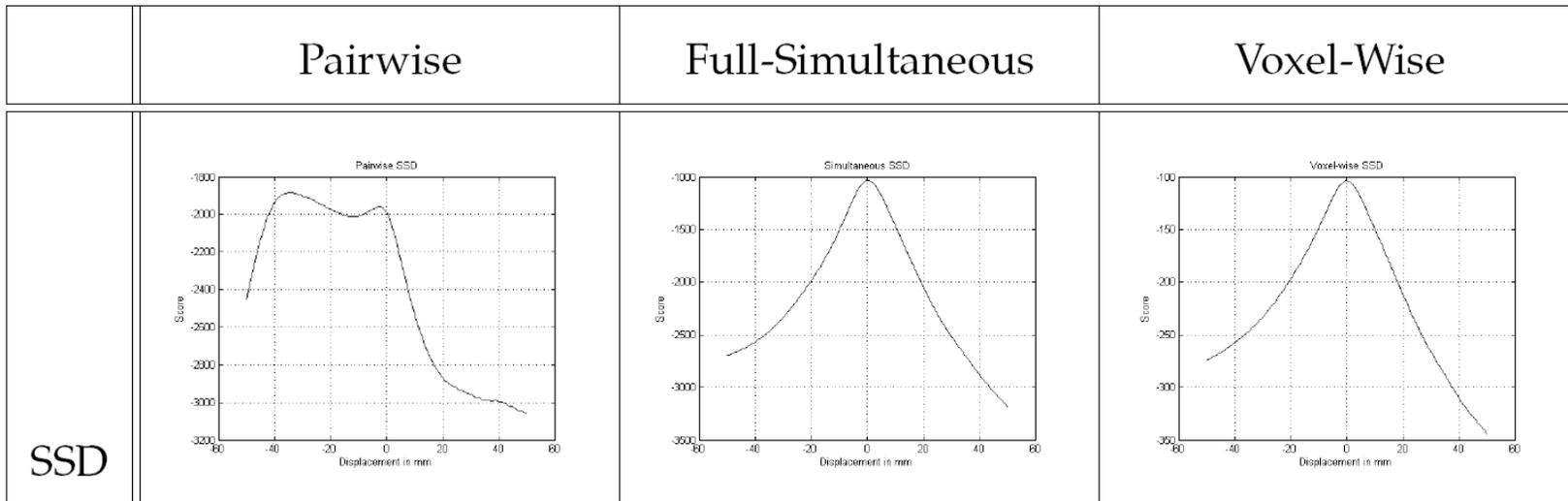


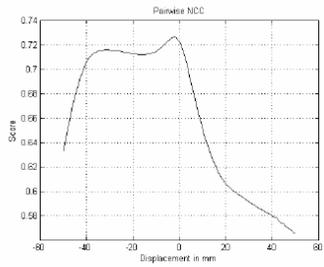
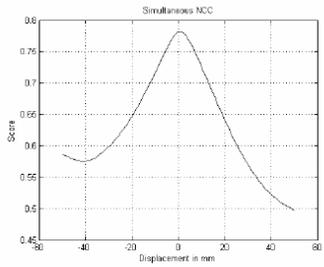
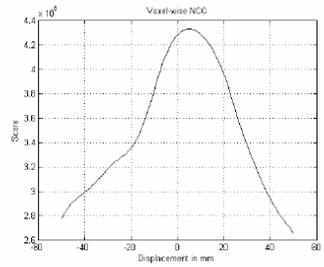
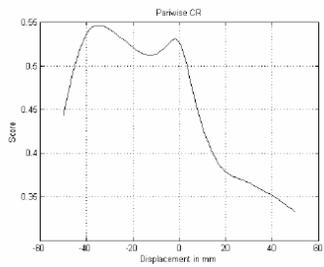
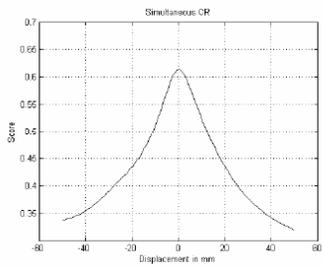
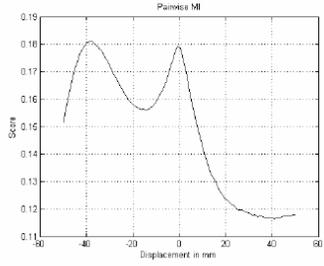
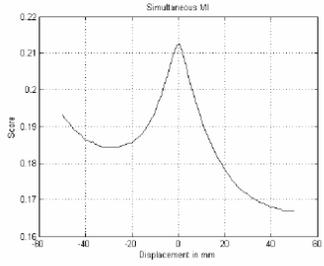
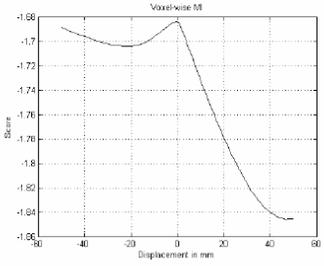
- Random study on abdominal phantom
  - 8 images
  - Up to  $\pm 20$  mm/degree random initial displacement
  - 100 registrations
  - Sum of Squared Differences
  - Plotting mean and standard deviation









	Pairwise	Full-Simultaneous	Voxel-Wise
NCC			
CR			<p style="text-align: center;">-</p>
MI			

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# Conclusion

- Ultrasound mosaicing as multiple image alignment
- Proposal of specific registration strategies for mosaicing
- Deduction of multivariate extensions for similarity measures under usage of a maximum likelihood framework
- Experiments show the superior performance of proposed strategies

	Sequential PW	Complete PW	Semi-Simultaneous	Full-Simultaneous
Optimization	6			
Similarity	Bivariate			
Overlap	Sequence			

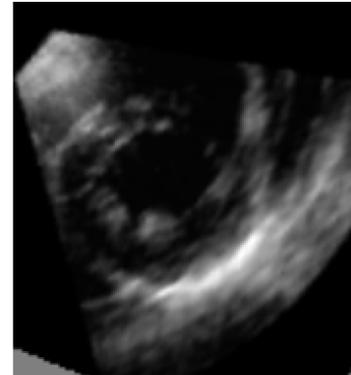
	Pairwise	Semi-Simultaneous	Full-Simultaneous	Voxel-Wise
SSD	$\mathbb{E}[(u - v^\perp)^2]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[(u_1 - v_i^\perp)^2]$		
NCC	$\mathbb{E}[\tilde{u} \cdot \tilde{v}^\perp]$	$\sum_{i=2}^n \omega_{1,i} \mathbb{E}[\tilde{u}_1 \cdot \tilde{v}_i^\perp]$		
CR	$\frac{\text{Var}[\mathbb{E}(u v^\perp)]}{\text{Var}(u)}$	$\sum_{i=2}^n \omega_{1,i} \frac{\text{Var}[\mathbb{E}(u_1 v_i^\perp)]}{\text{Var}(u_1)}$		
MI	$\text{MI}(u, v^\perp)$	$\sum_{i=2}^n \omega_{1,i} \text{MI}(u_1, v_i^\perp)$		

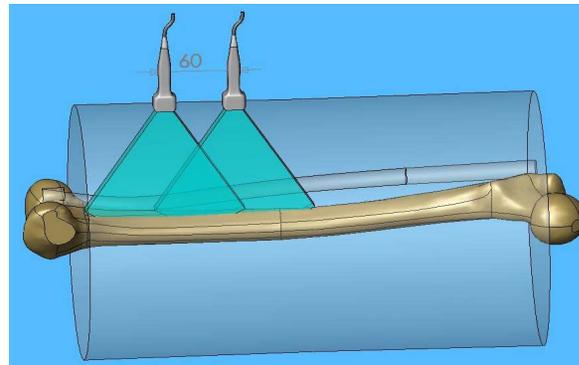
	Pairwise	Full-Simultaneous	Voxel-Wise
SSD			

# Further Development

- Integrating phase information
- Model-based registration
- Stochastic optimization
- Non-rigid registration
- Parallelization



Grau et al., Miccai '05 & '06



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The End