3D Computer Vision II

Reminder
Camera Models

Nassir Navab

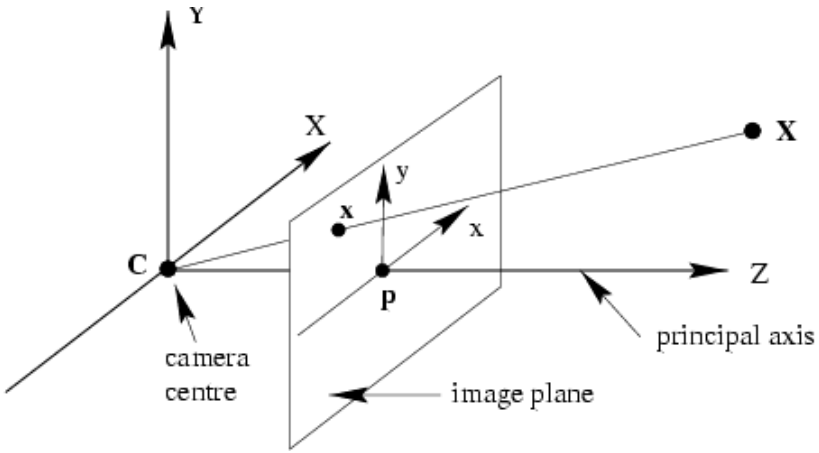
based on a course given at UNC by Marc Pollefeys & the book “Multiple View Geometry” by Hartley & Zisserman

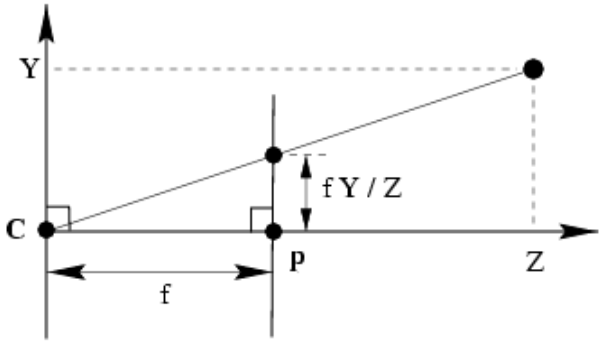
October 27, 2009

Outline – Camera Models

- Geometric Parameters of a Finite Camera
- Projective Camera Model
  - Camera Center, Principal Plane, Axis Plane, Principal Point, Principal Ray
  - Decomposition of Camera Matrix
- Cameras at Infinity
- Other Camera Models (Pushbroom and Line Cameras)

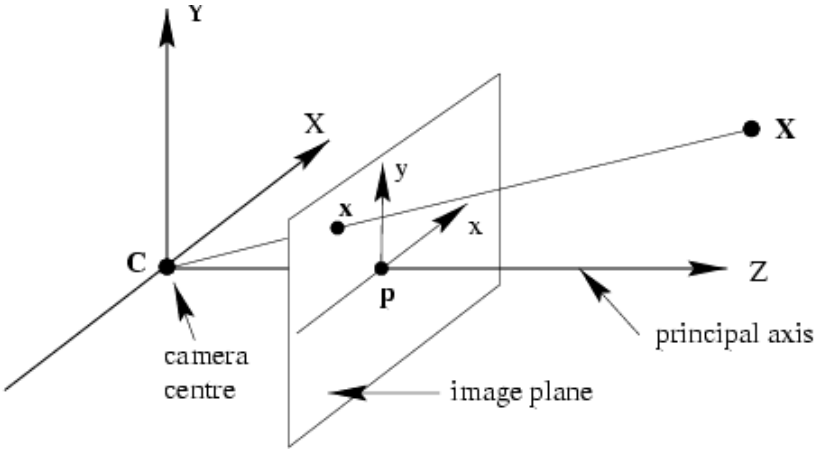
Pinhole Camera Model

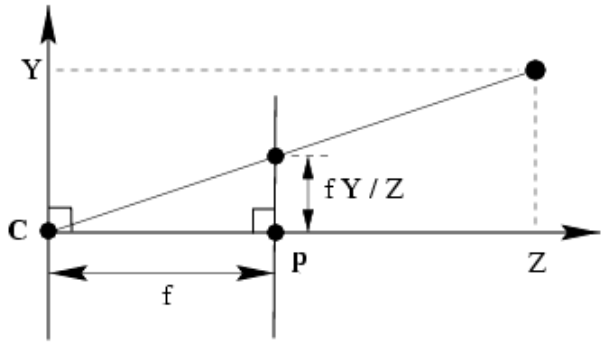




- Mapping between 3D world and 2D image $(X, Y, Z)^T \mapsto (x, y)^T$
- Central projection
- Models are described in matrices with particular properties

Homogeneous Coordinates

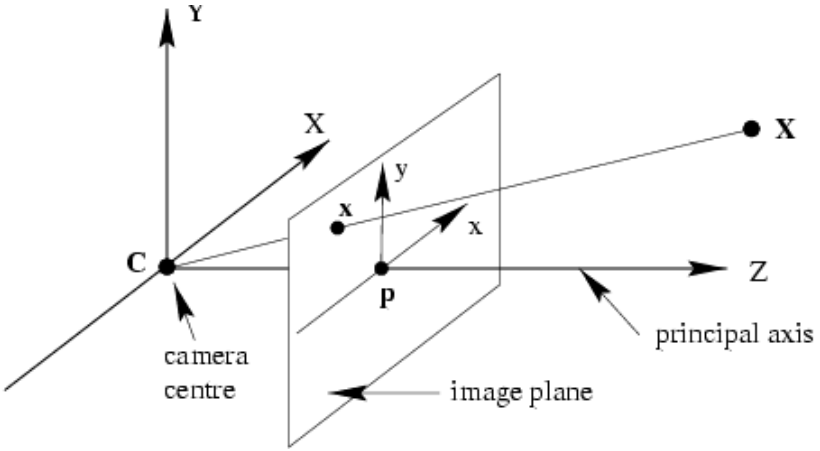


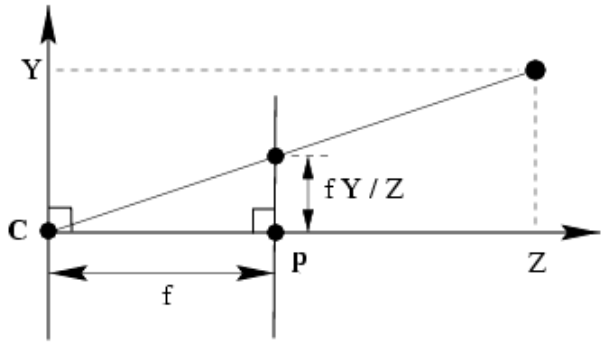


$$(X, Y, Z, 1)^T \mapsto (x, y, z)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Central Projection

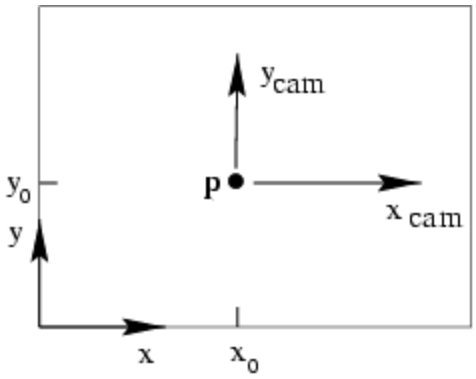


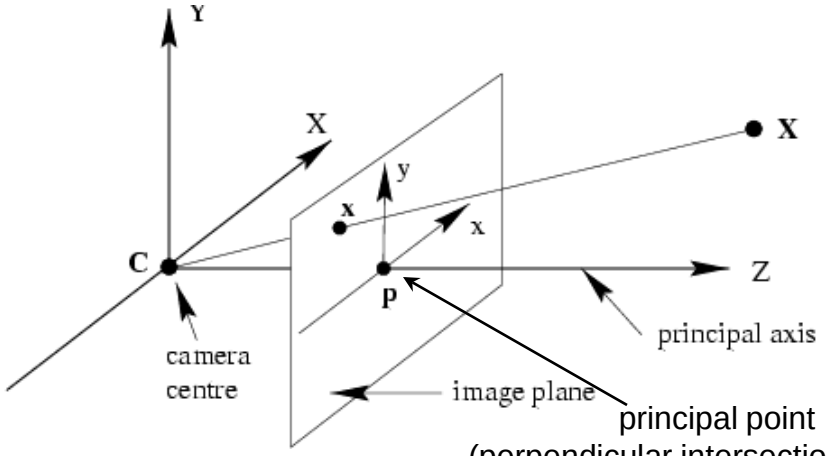


$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} = P\mathbf{X} \qquad P = \mathrm{diag}(f,f,1)\,[I\,|\,0]$$

Principal Point Offset





principal point
(perpendicular intersection point of principal axis and image plane)

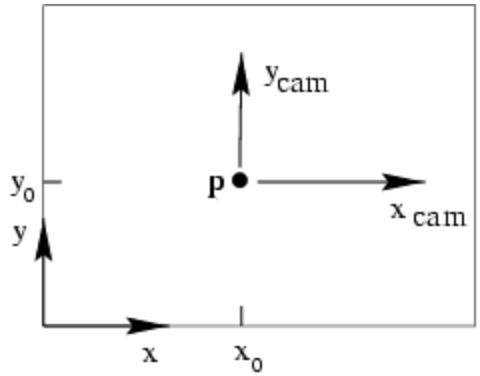
$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

where $(p_x, p_y)^T$

are the coordinates of the principal point

Principal Point Offset

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



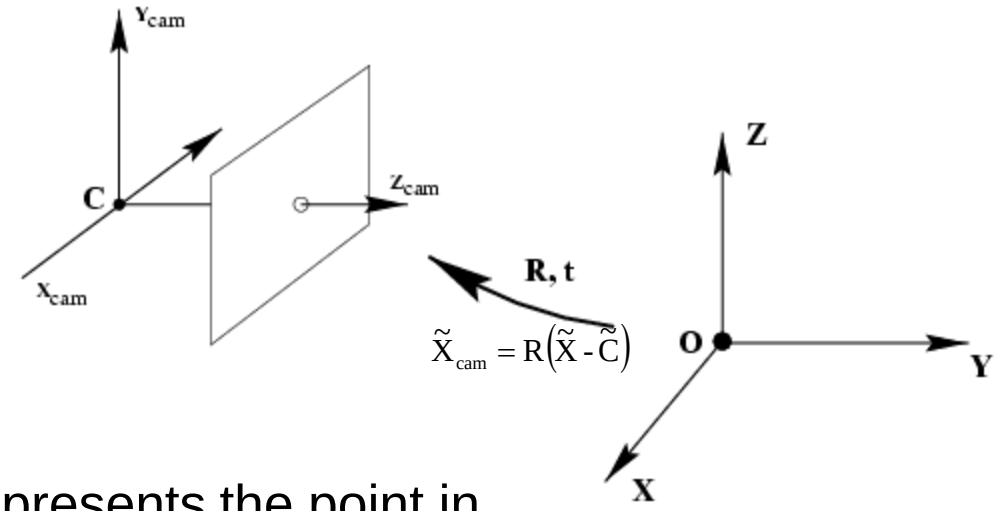
$$\mathbf{x} = K[I \mid 0]\mathbf{X}$$

where $K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$ is called camera calibration matrix

Camera Rotation and Translation

Inhomogeneous coordinates

$$\tilde{\mathbf{X}}_{\text{cam}} = \mathrm{R}\left(\tilde{\mathbf{X}} - \tilde{\mathrm{C}}\right)$$



where $\tilde{\mathbf{X}} = \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{pmatrix}$ represents the point in world coordinates

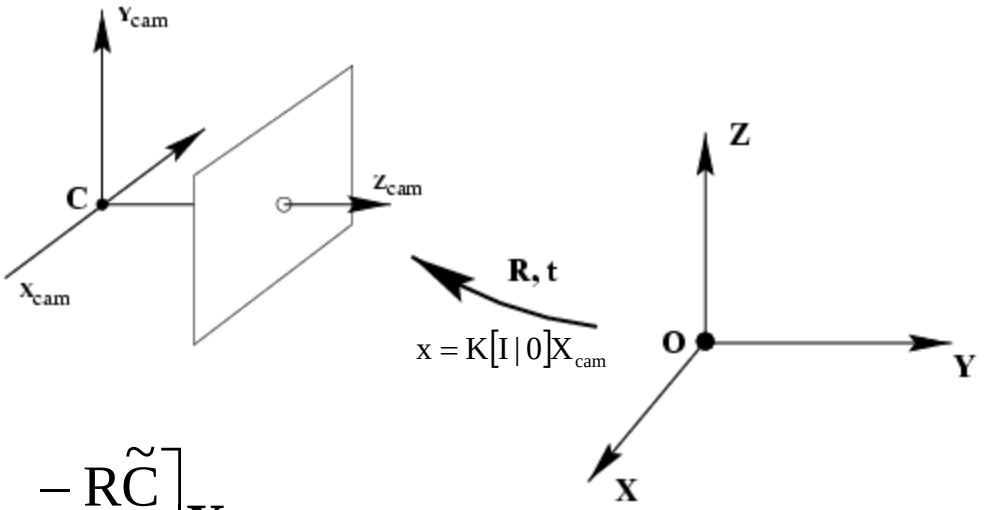
$\tilde{\mathbf{X}}_{\text{cam}} = \begin{pmatrix} \mathrm{x}_{\text{cam}} \\ \mathrm{y}_{\text{cam}} \\ \mathrm{z}_{\text{cam}} \end{pmatrix}$ represents the same point in camera coordinates

$\tilde{C}$ represents the coordinates of the camera origin in the world coordinate frame

Camera Rotation and Translation

Homogeneous coordinates

$$\mathbf{X}, \mathbf{X}_{cam} \in \mathrm{R}^4$$



$$\mathbf{X}_{\text{cam}} = \begin{bmatrix} \mathrm{R} & -\mathrm{R}\tilde{\mathrm{C}} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix} = \begin{bmatrix} \mathrm{R} & -\mathrm{R}\tilde{\mathrm{C}} \\ 0 & 1 \end{bmatrix} \mathbf{X}$$

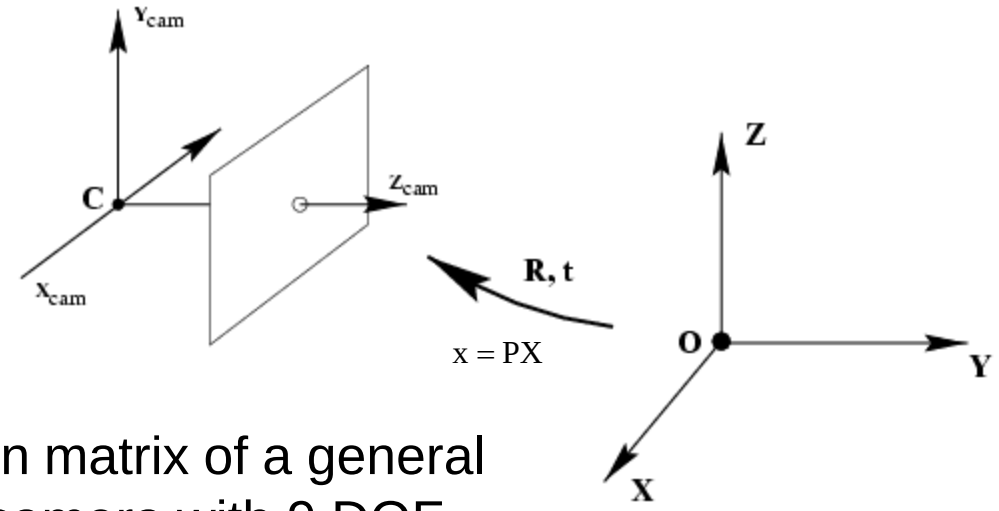
$\mathbf{x} = \mathrm{K}[\mathrm{I} \mid 0]\mathbf{X}_{\text{cam}}$ projection to image plane from camera coordinates

$\mathbf{x} = \mathrm{KR}[\mathrm{I} \mid -\tilde{\mathrm{C}}]\mathbf{X}$ projection to image plane from world coordinates

$\mathrm{x} = \mathrm{PX}$

Extrinsic and Intrinsic Parameters

$$x = PX$$



where

$$P = KR\left[I \mid -\tilde{C}\right]$$

projection matrix of a general pinhole camera with 9 DOF

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

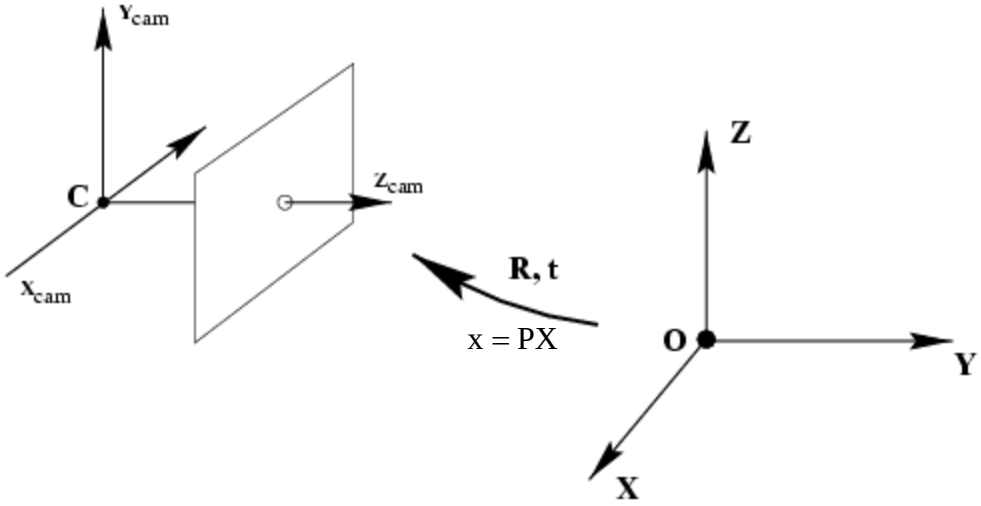
intrinsic camera parameters with 3 DOF

$$R, \tilde{C}$$

extrinsic camera parameters with each 3 DOF
(camera orientation and position in world coordinates)

Camera Rotation and Translation

No explicit
camera center



$$\tilde{\mathbf{X}}_{cam} = R\tilde{\mathbf{X}} + t$$

$$\mathrm{P} = \mathrm{K}\left[\mathrm{R} \mid \mathrm{t}\right]$$

$$= \mathrm{K}\left[\mathrm{R} \mid -\mathrm{R}\tilde{\mathrm{C}}\right]$$

where

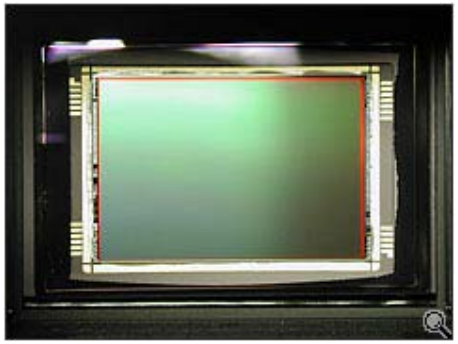
$\mathrm{t} = -\mathrm{R}\tilde{\mathrm{C}}$ from $\mathrm{x} = \mathrm{KR}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]\mathrm{X}$

CCD Cameras – Non-Square Pixels

m_x, m_y number of pixels per unit distance

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha_x & & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix}$$ 4 DOF



$$P = KR\left[I \mid -\tilde{C}\right]$$ 10 DOF

Skew Parameter

s skew parameter

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix}$$

5 DOF

$$P = KR\left[I \mid -\tilde{C}\right]$$

finite projective camera with
11 DOF

Finite Projective Camera – Summary

$$\mathbf{x} = \mathrm{P}\mathbf{X}$$

$$\mathrm{P} = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{R} & -\mathrm{R}\tilde{\mathrm{C}} \\ 0 & 1 \end{bmatrix}$$

$$= \underbrace{\mathrm{KR}}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]$$

non-singular

projection matrix
11 DOF (5+3+3)

Finite Projective Camera – Decomposition of P

$$M = KR$$

non-singular 3x3 matrix (8 DOF)

$$P = KR\left[I \mid -\tilde{C}\right]$$

$$P = M\left[I \mid -\tilde{C}\right]$$

decompose projection matrix P in K,R,C

$$P = \left[M \mid \mathbf{p}_4\right]$$

$$\left[K,R\right] = \underbrace{RQ}(M)$$

RQ matrix decomposition

$$\tilde{C} = -M^{-1}\mathbf{p}_4$$

Finite Projective Camera – Summary

$$P = KR\left[I \mid -\tilde{C}\right] = M\left[I \mid -\tilde{C}\right]$$

where $P \in R^{3\times 4}$

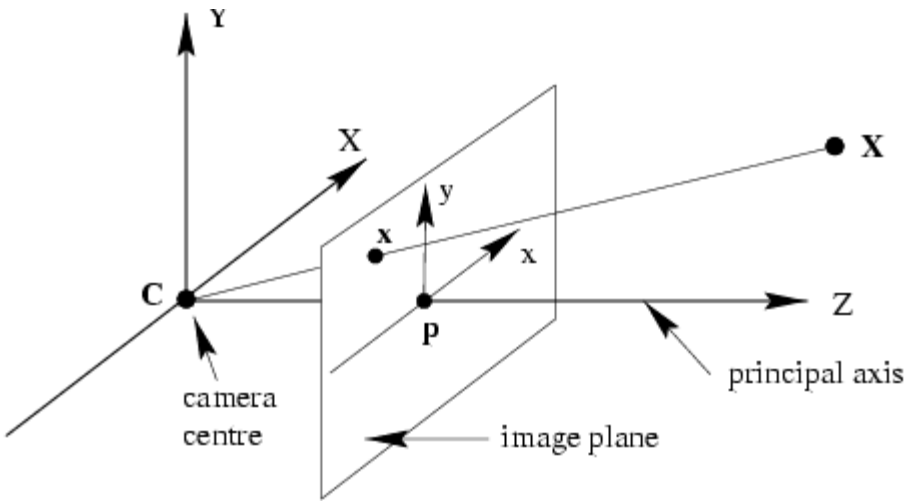
- Camera matrices P are identical with the set of homogeneous 3x4 matrices for which the left 3x3 sub-matrix is non-singular
- {finite cameras}={P| det M≠0} ={P| rank(M)=3}
- If rank(P)=3, but rank(M)<3, then camera at infinity
- if rank(P)<3 the matrix mapping will be a line or a point and not a plane (not a 2D image)

Outline – Camera Models

- Geometric Parameters of a Finite Camera
- Projective Camera Model
  - Camera Center, Principal Plane, Axis Plane, Principal Point, Principal Ray
  - Decomposition of Camera Matrix
- Cameras at Infinity
- Other Camera Models (Pushbroom and Line Cameras)

Camera Anatomy

- Camera center
- Column vectors
- Principal plane
- Axis plane
- Principal point
- Principal ray



Camera Center

$$PC = 0$$ P has a 1D null-space

we will prove that the 4-vector C is the camera center

$X(\lambda) = \lambda A + (1-\lambda)C$ points on a line through A and C

$$x = PX = \lambda PA + (1-\lambda)PC = \lambda PA$$ since $PC = 0$

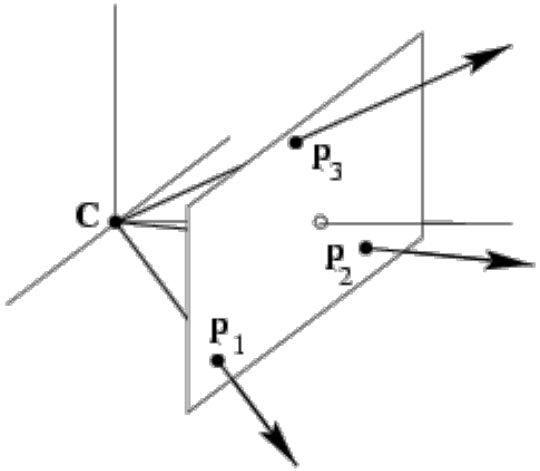
All 3D points on the line are mapped on the same 2D image point, and thus the line is a ray through the camera center

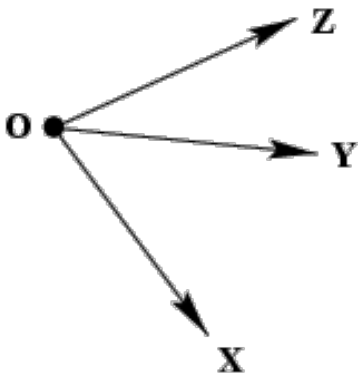
Finite cameras: $$C = \begin{pmatrix} \tilde{C} \\ 1 \end{pmatrix} = \begin{pmatrix} -M^{-1}\mathbf{p}_4 \\ 1 \end{pmatrix}$$

Infinite cameras: $$C = \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix}, M\mathbf{d} = 0$$

Column Vectors

$$\mathbf{p}_i \in \mathrm{R}^3, i = 1,...,4$$





Column vectors are the image points which project the axis directions (X,Y,Z) and the origin

Example for the image of the y-axis

$$[\mathbf{p}_2] = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \quad \mathbf{p}_4][0 \quad 1 \quad 0 \quad 0]^T$$

$\mathbf{p}_4$ is the image of the world origin

Row Vectors

$$\mathbf{p}^i \in \mathrm{R}^4, i = 1,...,3$$

Represent geometrically particular world planes.

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$$

row vectors

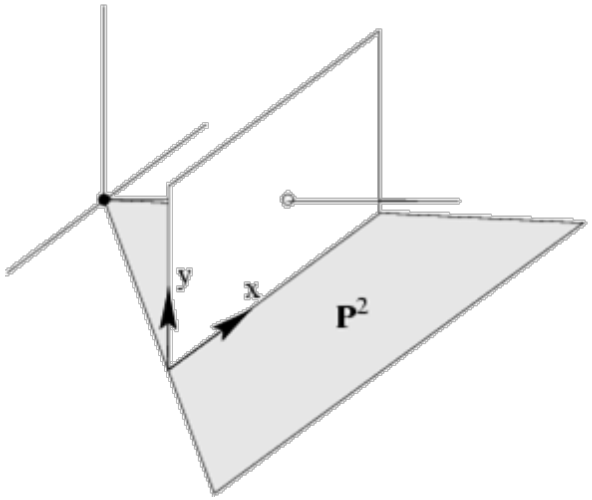
$$= \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix}$$

column vectors

Row Vectors of the Projection Matrix

P^1 is defined by the camera center and the line x=0 on the image.

P^2 is defined by the camera center and the line y=0 on the image.



Example P^2

$$\begin{bmatrix} x \\ 0 \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{1^T} \\ \mathbf{p}^{2^T} \\ \mathbf{p}^{3^T} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

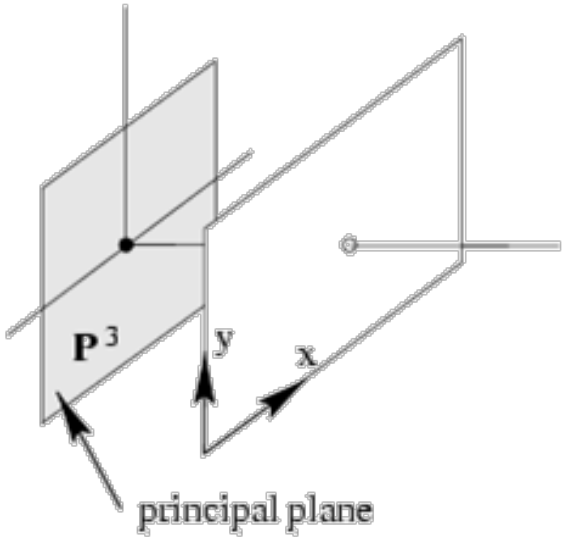
$$\mathbf{p}^{2^T}\mathbf{X} = 0$$

$$\mathbf{p}^{2^T}C = 0$$

respectively for P^1

Principal Plane

Plane through camera center and parallel to the image plane.



$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{p}^{1\mathsf{T}} \\ \mathrm{p}^{2\mathsf{T}} \\ \mathrm{p}^{3\mathsf{T}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

points **X** are imaged on the line at infinity

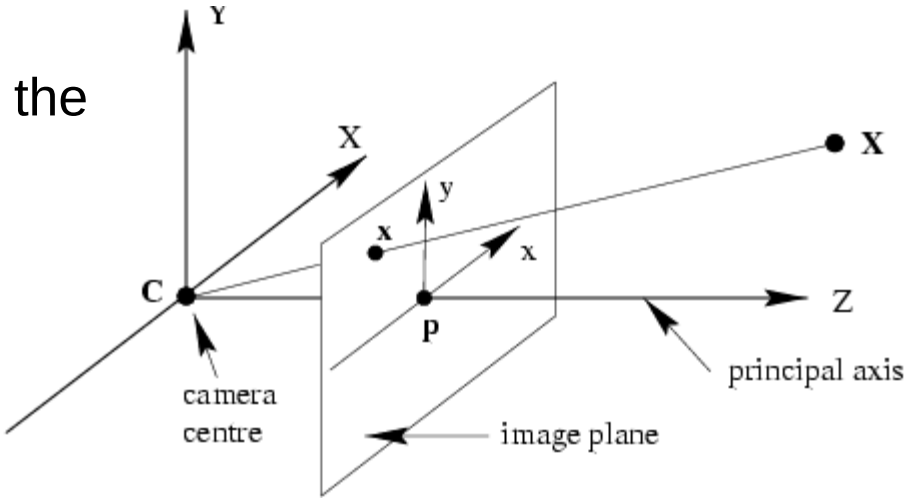
if **X** is on the principle plane $\mathbf{p}^{3T}\mathbf{X} = 0$

especially $\mathbf{p}^{3T}\mathrm{C} = 0$

Principal Point

Line through camera center and perpendicular to principal plane is the principal axis.

Intersection of the principal axis with the image plane is the principal point.



$\hat{\mathbf{P}}^3 = \begin{pmatrix} p_{31} & p_{32} & p_{33} & 0 \end{pmatrix}^{\mathrm{T}}$ normal direction to principal plane

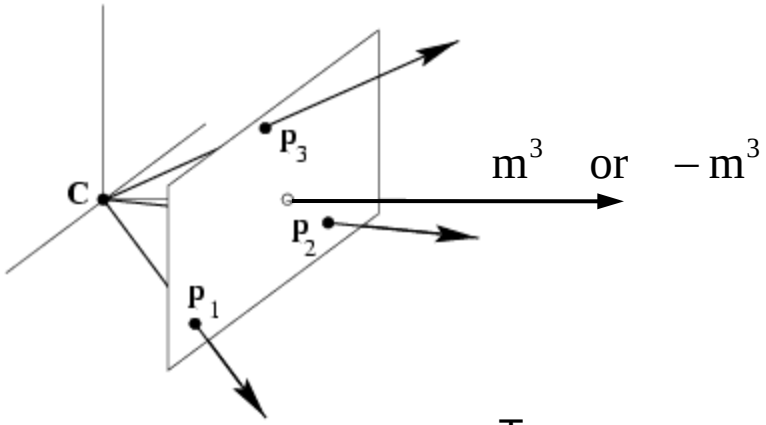
principal point

$$\mathrm{x}_0 = \mathrm{P}\hat{\mathbf{P}}^3 = \mathrm{M}\mathbf{m}^3 \quad \text{where} \quad \mathrm{P} = \left[\mathrm{M} \mid \mathbf{p}_4\right]$$

and $\mathbf{m}^{3^{\mathrm{T}}}$ third row of M

Principal Axis Vector

Ambiguity that principal axis points towards the front of the camera (positive direction)



$$x = P_{cam} X_{cam} = K[I \mid 0] X_{cam}$$

$$v = \det(K) k^3 = (0,0,1)^T$$

towards the front of the camera

$$P_{cam} \mapsto kP_{cam} \qquad v \mapsto k^4 v$$

direction unaffected by scaling

$$P = kKR[I \mid -\tilde{C}] = [M \mid p_4]$$

since $\det(R) > 0$

$$v = \det(M) m^3$$

Forward Projection

Maps a point in space on the image plane $\mathbb{P}^3 \rightarrow \mathbb{P}^2$

$$x = PX$$

Vanishing points $D = \left(d^T, 0\right)^T$

$$x = PD = \left[M \mid p_4\right]D = Md$$

Only M affects the projection of vanishing points

Back-Projection to Rays

Points on the reconstructed ray

$$PC = 0$$ camera center C

$$X = P^{+}x \qquad P^{+} = P^{T}\left(PP^{T}\right)^{-1} \qquad PP^{+} = I$$

(pseudo-inverse)

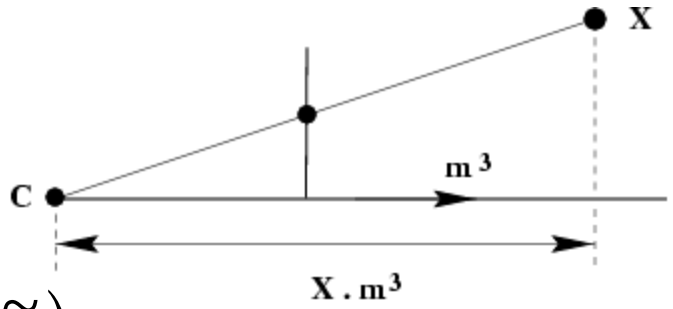
Ray is the line formed by those two points

$$X(\lambda) = P^{+}x + \lambda C$$

$$D = \begin{pmatrix} \left(M^{-1}x\right)^{T}, 0 \end{pmatrix}^{T}$$ intersection of the ray with the plane at infinity

$$X(\mu) = \mu \underbrace{\begin{pmatrix} M^{-1}x \\ 0 \end{pmatrix}}_{D} + \underbrace{\begin{pmatrix} -M^{-1}p_4 \\ 1 \end{pmatrix}}_{C} = \begin{pmatrix} M^{-1}(\mu x - p_4) \\ 1 \end{pmatrix}$$

Depth of Points



$$w = \mathrm{P}^{3\mathrm{T}}\mathrm{X} = \mathrm{P}^{3\mathrm{T}}\left(\mathrm{X} - \mathrm{C}\right) = \mathrm{m}^{3\mathrm{T}}\left(\tilde{\mathrm{X}} - \tilde{\mathrm{C}}\right)$$

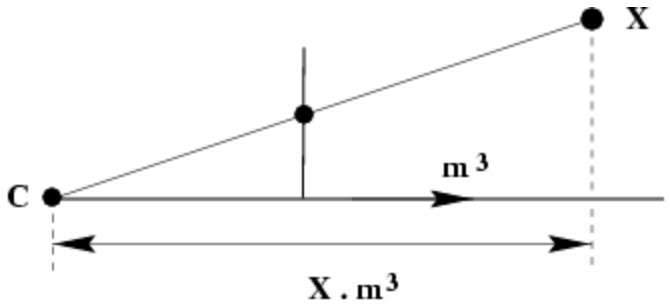
(PC=0) (dot product)

If $\det M > 0; \left\|\mathrm{m}^3\right\| = 1,$
then m^3 unit vector pointing in positive axis direction

Suppose $P\left(X,Y,Z,T\right)^{\mathrm{T}} = w(x, y, 1)^{\mathrm{T}}$. Then

$$\mathrm{depth}\left(\mathrm{X};\mathrm{P}\right) = \frac{\mathrm{sign}(\det \mathrm{M})w}{T\left\|\mathrm{m}^3\right\|}$$

Depth of Points: Examples



$$\text{depth}(\mathrm{X};\mathrm{P}) = \frac{\text{sign}(\det \mathrm{M})\, w}{\left\| \mathrm{m}^3 \right\|}$$

$$P(X,Y,Z,1)^{\mathrm{T}} = w(x,y,1)^{\mathrm{T}}$$

$$\mathbf{x} = \mathrm{P}\mathbf{X}$$

$$\mathrm{P} = [\mathrm{I} \,|\, 0]$$

$$depth(X;P) = Z$$

$$\mathbf{x} = \mathrm{P}\mathbf{X}$$

$$\mathrm{P} = \mathrm{R}[\mathrm{I} \,|\, -\tilde{\mathrm{C}}]$$

$$\text{depth}(\mathrm{X};\ \mathrm{P}) \ = \ \mathrm{R}_3^{\,T}(\tilde{X} - \tilde{\mathrm{C}})$$

Outline – Camera Models

- Geometric Parameters of a Finite Camera
- Projective Camera Model
  - Camera Center, Principal Plane, Axis Plane, Principal Point, Principal Ray
  - Decomposition of Camera Matrix
- Cameras at Infinity
- Other Camera Models (Pushbroom and Line Cameras)

Camera Matrix Decomposition

Finding the camera center C

$$\mathrm{PC} = 0$$

numerically: find right null-space by SVD of P

$$\mathrm{P}^{3\mathrm{x}4} = \mathrm{U}^{3\mathrm{x}4}\mathrm{D}^{4\mathrm{x}4}\mathrm{V}^{4\mathrm{x}4\mathrm{T}} \rightarrow \mathrm{P}^{3\mathrm{x}4}\mathrm{V}^{4\mathrm{x}4} = \mathrm{U}^{3\mathrm{x}4}\mathrm{D}^{4\mathrm{x}4} \rightarrow \mathrm{PV}^{4} = 0$$

Algebraically:

$$X = \det\left(\left[\mathrm{p}_2, \mathrm{p}_3, \mathrm{p}_4\right]\right) \quad Y = -\det\left(\left[\mathrm{p}_1, \mathrm{p}_3, \mathrm{p}_4\right]\right)$$
$$Z = \det\left(\left[\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_4\right]\right) \quad T = -\det\left(\left[\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3\right]\right)$$

where $\mathrm{C} = \left(X, Y, Z, T\right)^{\mathrm{T}}$

Camera Matrix Decomposition

Finding the camera center C

$$\mathrm{PC} = 0$$

Any plane π going through C will be a linear combination of the three planes defined by the rows of P. Therefore:

$$\det\left(\left[\pi,\ \mathrm{P}^{\mathrm{T}}\right]\right) = 0$$

$$= \pi_1 \det(P_{234}) - \pi_2 \det(P_{134}) + \pi_3 \det(P_{124}) - \pi_4 \det(P_{123})$$

$$X = \det\left(\left[\mathrm{p}_2, \mathrm{p}_3, \mathrm{p}_4\right]\right) \quad Y = -\det\left(\left[\mathrm{p}_1, \mathrm{p}_3, \mathrm{p}_4\right]\right)$$

$$Z = \det\left(\left[\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_4\right]\right) \quad T = -\det\left(\left[\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3\right]\right)$$

where $\mathrm{C} = \left(X, Y, Z, T\right)^{\mathrm{T}}$

Camera Matrix Decomposition

Finding the camera orientation and internal parameters

$$P = [M \mid -M\tilde{C}] = K[R \mid -R\tilde{C}]$$

$$K = \begin{bmatrix} \alpha_x & s & y_0 \\ 0 & \alpha_y & x_0 \\ 0 & 0 & 1 \end{bmatrix}$$

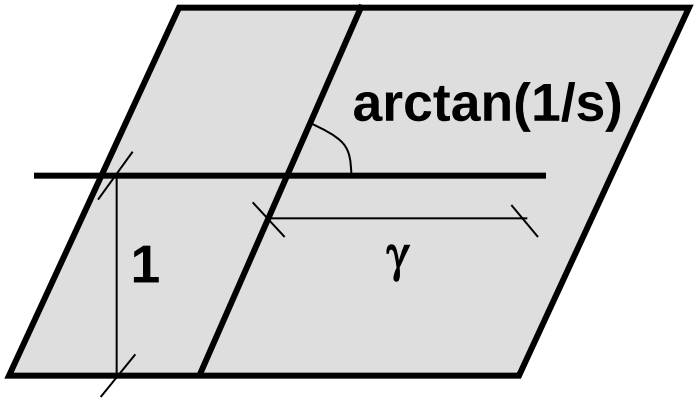
Decompose $M = KR$ using RQ decomposition

$$\square = (\boxed{Q}\ \triangleleft R)^{-1} = \triangleleft R^{-1}\ \boxed{Q}^{-1}$$

Ambiguity removed by enforcing positive diagonal entries

When is Skew Non-zero?

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix}$$



for CCD/CMOS, always s=0

Image from image, s≠0 possible
(non coinciding principal axis)

resulting camera: HP

where H is a 3x3 homography

Euclidean vs. Projective Spaces

General projective interpretation

$$P = [3 \times 3 \text{ homography}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} [4 \times 4 \text{ homography}]$$

- Meaningful decomposition in K,R,t requires Euclidean image and space
- Camera center is still valid in projective space
- Principal plane requires affine image and space
- Principal ray requires affine image and Euclidean space

Outline – Camera Models

- Geometric Parameters of a Finite Camera
- Projective Camera Model
  - Camera Center, Principal Plane, Axis Plane, Principal Point, Principal Ray
  - Decomposition of Camera Matrix
- Cameras at Infinity
- Other Camera Models (Pushbroom and Line Cameras)

Cameras at Infinity

Cameras with their center lying at infinity

$$P\begin{bmatrix} d \\ 0 \end{bmatrix} = 0 \Rightarrow \det M = 0$$ M is singular

Two types of cameras at infinity:
Affine and non-affine cameras

Affine Cameras

Definition:
An affine camera is a camera with a camera matrix P in which the last row p^{3T} is of the form $(0,0,0,1)^T$.

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Points at infinity are mapped to points at infinity

Parallel Projections

$$P_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

canonical representation

$$K = \begin{bmatrix} K_{2\times 2} & 0 \\ 0 & 1 \end{bmatrix}$$

calibration matrix

principal point is not defined

Hierarchy of Affine Cameras

$$P_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

dropping the z-coordinate

orthographic projection

$$P_\infty = \begin{bmatrix} r^{1T} & t_1 \\ r^{1T} & t_2 \\ 0 & 1 \end{bmatrix}$$

(5dof)

Hierarchy of Affine Cameras

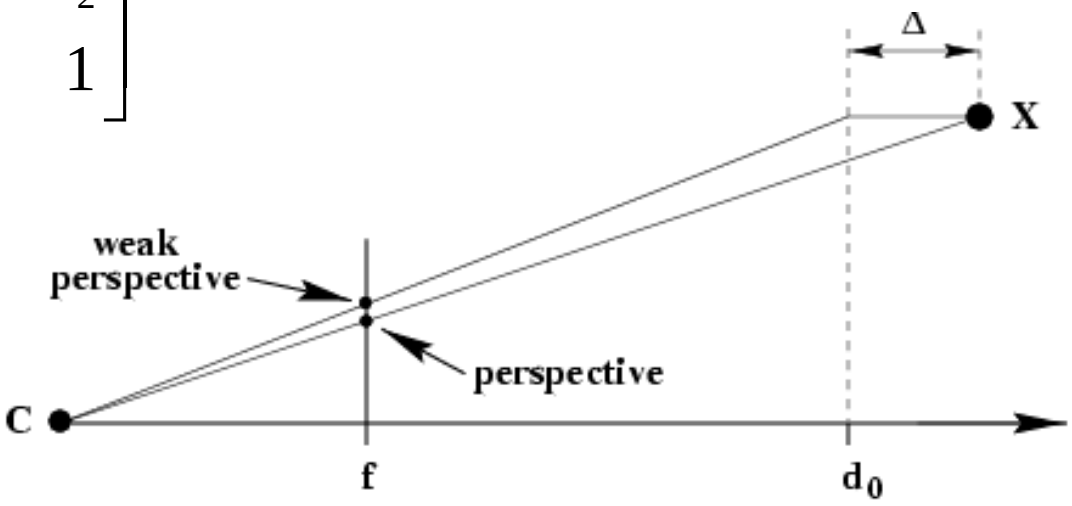
scaled orthographic projection

$$\mathrm{P}_\infty = \begin{bmatrix} k & & \\ & k & \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}^{1\mathrm{T}} & t_1 \\ \mathrm{r}^{1\mathrm{T}} & t_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathrm{r}^{1\mathrm{T}} & t_1 \\ \mathrm{r}^{1\mathrm{T}} & t_2 \\ 0 & 1/k \end{bmatrix} \quad \text{(6dof)}$$

Hierarchy of Affine Cameras

weak perspective projection

$$P_\infty = \begin{bmatrix} \alpha_x & & \\ & \alpha_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} r^{1T} & t_1 \\ r^{1T} & t_2 \\ 0 & 1 \end{bmatrix} \quad \text{(7dof)}$$



Hierarchy of Affine Cameras

Affine camera

$$P_A = \begin{bmatrix} \alpha_x & s & \\ & \alpha_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} r^{1T} & t_1 \\ r^{1T} & t_2 \\ 0 & 1 \end{bmatrix} \quad \text{(8dof)} \qquad P_A = \begin{bmatrix} m_{11} & m_{12} & m_{13} & t_1 \\ m_{21} & m_{22} & m_{23} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

full generality of an affine camera

$$P_A = [3 \times 3 \text{ affine}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [4 \times 4 \text{ affine}]$$

- Affine camera is a projective camera with principal plane at infinity
- Affine camera maps parallel world lines to parallel image lines
- No center of projection, but direction of projection $P_A D=0$

General Camera at Infinity

$$P = [M \mid p_4]$$ M is singular, but last row not zero

- Camera center is on plane at infinity
- Principal plane is not plane at infinity
- Images of points at infinity are in general not mapped to infinity on the image plane

Summary Camera Models

- Photometric and radiometric properties of a camera
- Geometric parameters of a finite camera
- Projective cameras
  - Camera anatomy (camera center, principle plane, principle point, and principle axis)
  - Camera matrix decomposition (camera center, orientation, and intrinsic parameter
- Cameras at infinity
  - Affine cameras
  - Non-affine cameras

Literature on Camera Models

- Chapter 6 in R. Hartley and A. Zisserman, "*Multiple View Geometry*", 2nd edition, Cambridge University Press, 2003.
- Chapter 3 in O. Faugeras, "Three-dimensional Computer Vision", MIT Press, 1993.
- Chapter 2 in E. Trucco and A. Verri, "*Introductory Techniques for 3-D Computer Vision*", Prentice Hall, 1998.
- H. Gernsheim, "The Origins of Photography", *Thames and Hudson*, 1982.
- A. Shashua. "*Geometry and Photometry in 3D Visual Recognition"*, Ph.D. Thesis, MIT, Nov. 1992. AITR-1401.