

3D Computer Vision II

Reminder Camera Models

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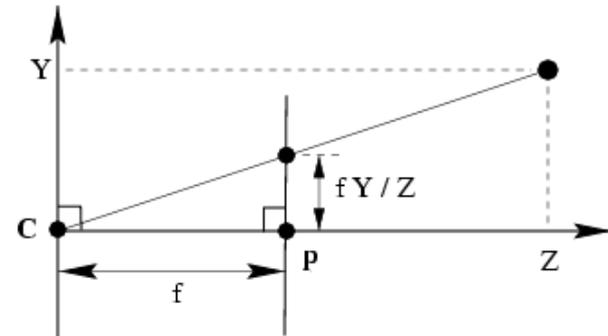
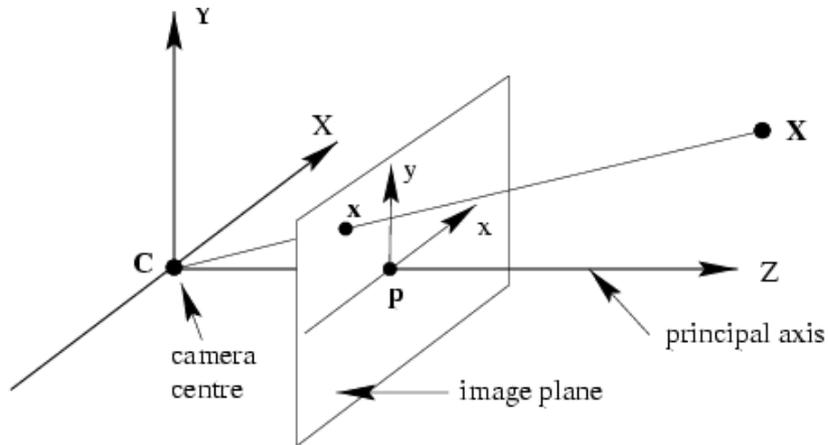
based on a course given at UNC by Marc Pollefeys & the book "Multiple View Geometry" by Hartley & Zisserman

October 21, 2010

Outline – Camera Models

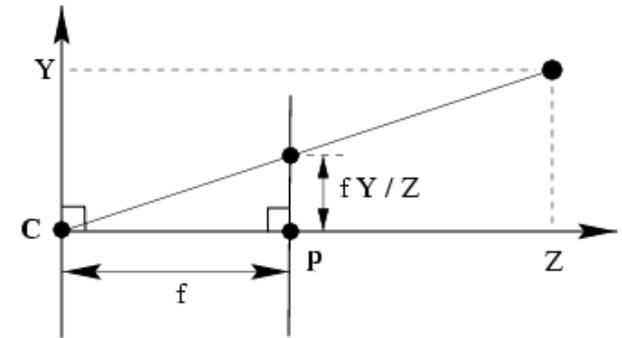
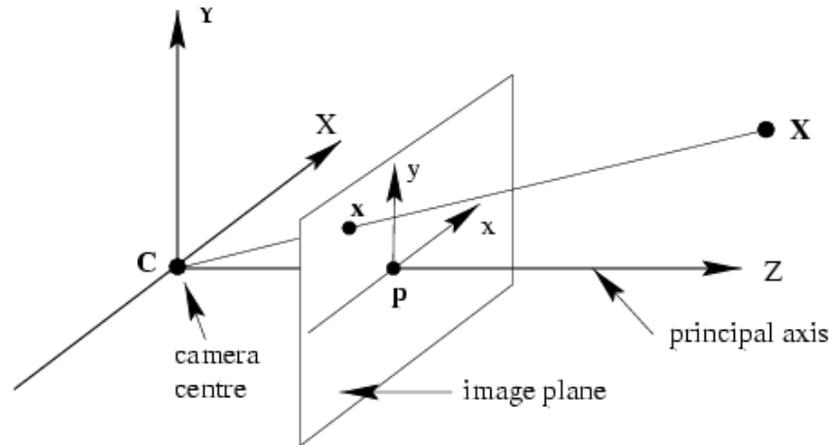
- Geometric Parameters of a Finite Camera
- Projective Camera Model
 - Camera Center, Principal Plane, Axis Plane, Principal Point, Principal Ray
 - Decomposition of Camera Matrix
- Cameras at Infinity
- Other Camera Models (Pushbroom and Line Cameras)

Pinhole Camera Model



- Mapping between 3D world and 2D image $(X, Y, Z)^T \mapsto (x, y)^T$
- Central projection
- Models are described in matrices with particular properties

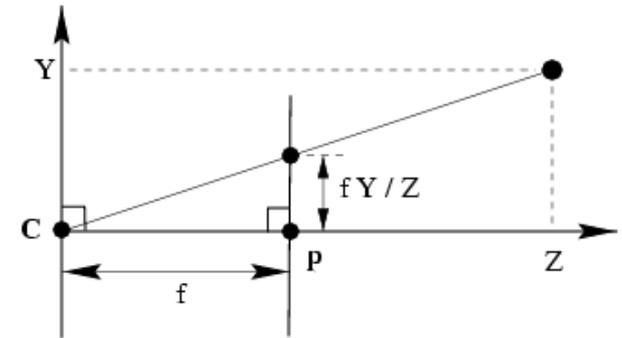
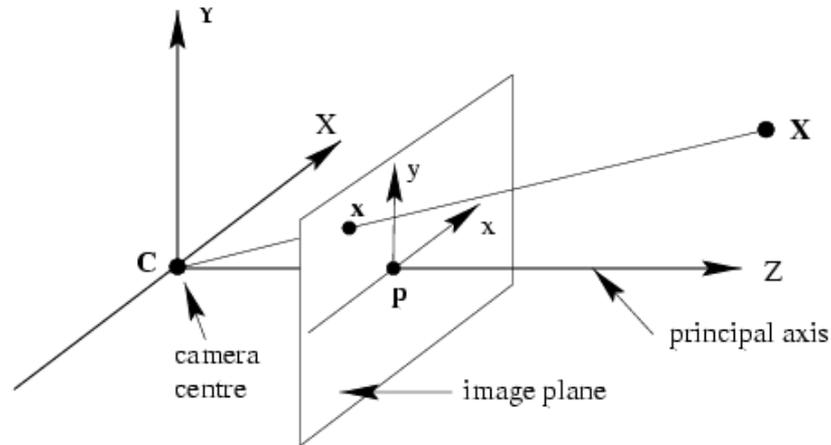
Homogeneous Coordinates



$$(X, Y, Z, 1)^T \mapsto (x, y, z)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

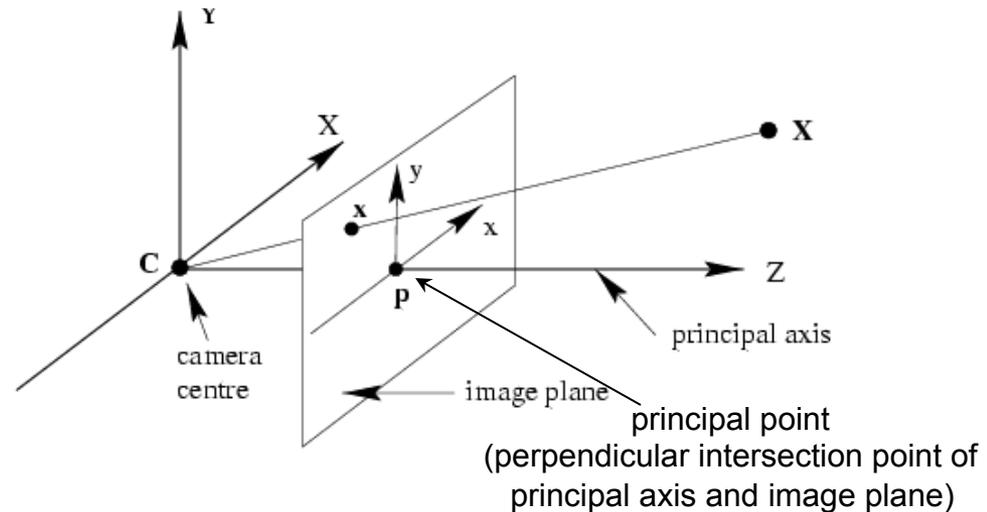
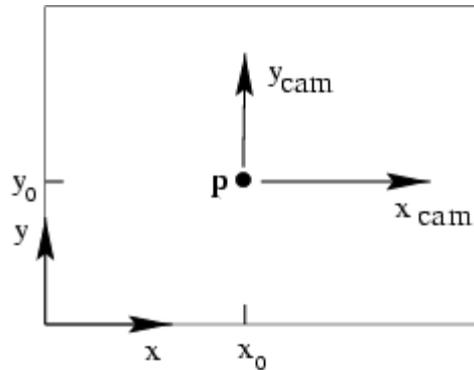
Central Projection



$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & \\ & f & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X} \quad \mathbf{P} = \text{diag}(f, f, 1) [\mathbf{I} \mid \mathbf{0}]$$

Principal Point Offset



$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

where $(p_x, p_y)^T$

are the coordinates of the principal point

Principal Point Offset

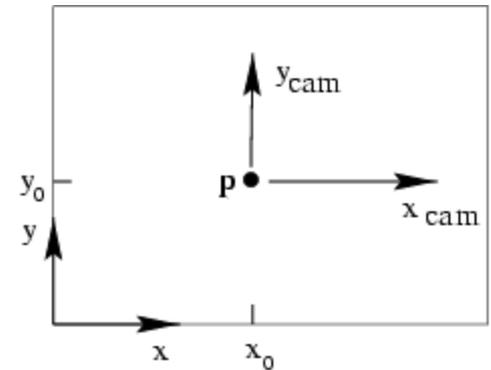
$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & p_x & 0 \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{K}[\mathbf{I} | \mathbf{0}] \mathbf{X}$$

where

$$\mathbf{K} = \begin{bmatrix} f & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

is called camera calibration matrix



Camera Rotation and Translation

Inhomogeneous coordinates

$$\tilde{\mathbf{X}}_{\text{cam}} = \mathbf{R}(\tilde{\mathbf{X}} - \tilde{\mathbf{C}})$$

where

$$\tilde{\mathbf{X}} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

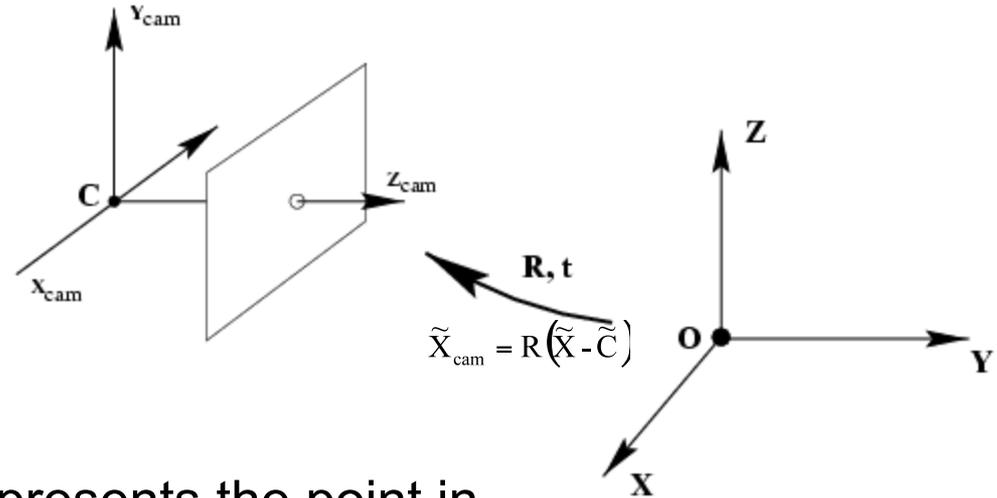
$$\tilde{\mathbf{X}}_{\text{cam}} = \begin{pmatrix} X_{\text{cam}} \\ y_{\text{cam}} \\ Z_{\text{cam}} \end{pmatrix}$$

$$\tilde{\mathbf{C}}$$

represents the point in world coordinates

represents the same point in camera coordinates

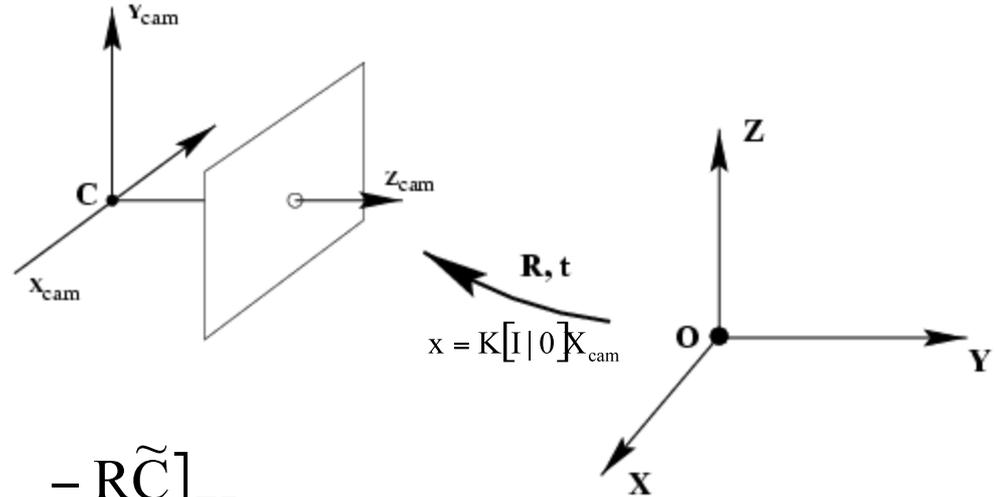
represents the coordinates of the camera origin in the world coordinate frame



Camera Rotation and Translation

Homogeneous coordinates

$$\mathbf{X}, \mathbf{X}_{cam} \in \mathbb{R}^4$$



$$\mathbf{X}_{cam} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix} \mathbf{X}$$

$\mathbf{x} = \mathbf{K}[\mathbf{I} | 0]\mathbf{X}_{cam}$ projection to image plane from camera coordinates

$\mathbf{x} = \mathbf{K}\mathbf{R}[\mathbf{I} | -\tilde{\mathbf{C}}]\mathbf{X}$ projection to image plane from world coordinates

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

Extrinsic and Intrinsic Parameters

$$x = PX$$

where

$$P = KR \begin{bmatrix} I & | & -\tilde{C} \end{bmatrix}$$

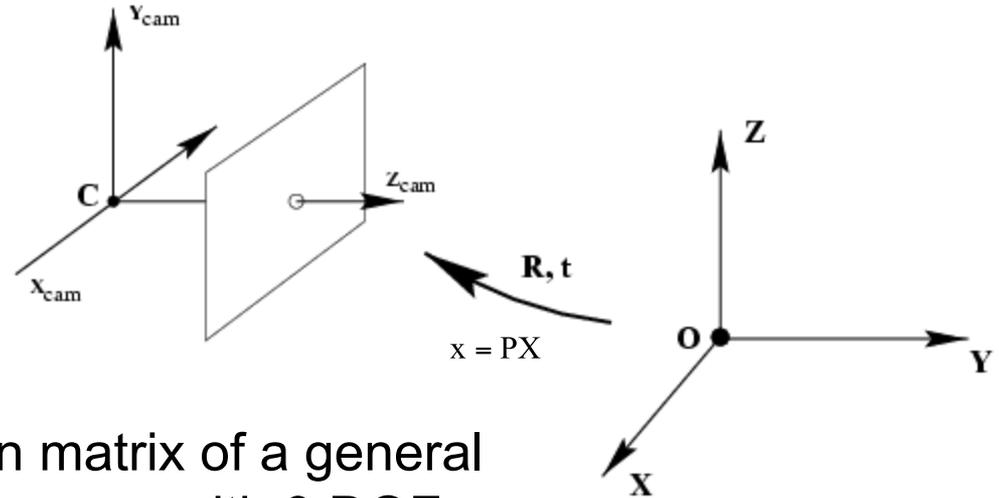
projection matrix of a general pinhole camera with 9 DOF

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

intrinsic camera parameters with 3 DOF

$$R, \tilde{C}$$

extrinsic camera parameters with each 3 DOF
(camera orientation and position in world coordinates)



Camera Rotation and Translation

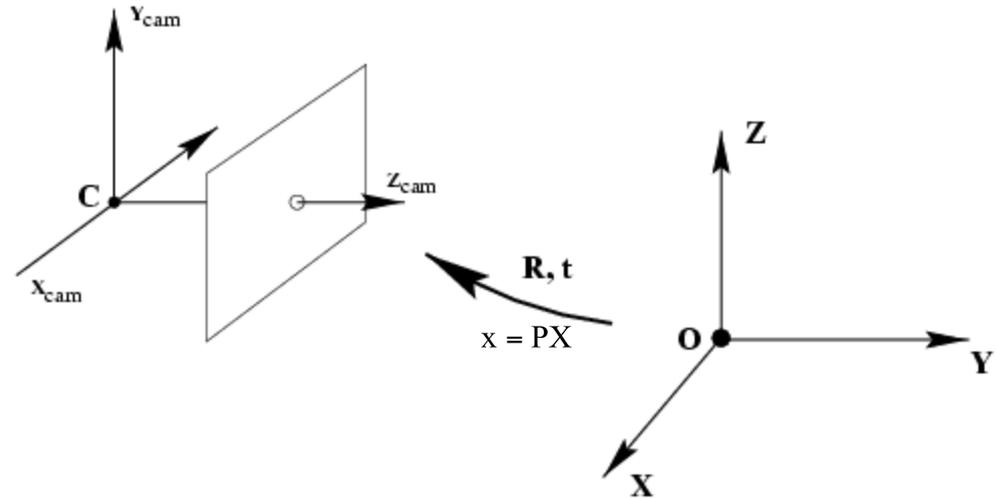
No explicit
camera center

$$\tilde{\mathbf{X}}_{cam} = R\tilde{\mathbf{X}} + t$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R} \mid \mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} \mid -\mathbf{R}\tilde{\mathbf{C}}] \end{aligned}$$

where

$$t = -R\tilde{\mathbf{C}} \quad \text{from} \quad \mathbf{x} = \mathbf{K}\mathbf{R}[\mathbf{I} \mid -\tilde{\mathbf{C}}]\mathbf{X}$$



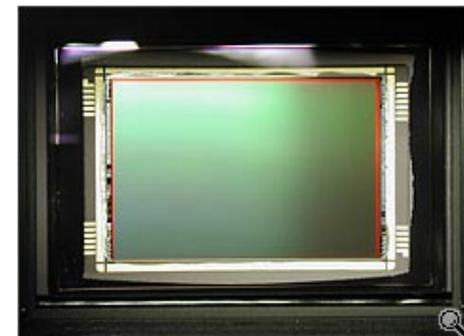
CCD Cameras – Non-Square Pixels

m_x, m_y number of pixels per unit distance

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & p_x \\ f & p_y \\ & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha_x & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix} \quad 4 \text{ DOF}$$

$$P = KR \begin{bmatrix} I & | & -\tilde{C} \end{bmatrix} \quad 10 \text{ DOF}$$



Skew Parameter

S skew parameter

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix} \quad 5 \text{ DOF}$$

$$P = KR \begin{bmatrix} I & -\tilde{C} \end{bmatrix} \quad \text{finite projective camera with 11 DOF}$$

Finite Projective Camera – Summary

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix} \quad \begin{array}{l} \text{projection matrix} \\ 11 \text{ DOF } (5+3+3) \end{array}$$

$$= \underbrace{\mathbf{KR}}_{\text{non-singular}} \left[\mathbf{I} \mid -\tilde{\mathbf{C}} \right]$$

non-singular

Finite Projective Camera – Decomposition of P

$$M = KR \quad \text{non-singular } 3 \times 3 \text{ matrix (8 DOF)}$$

$$P = KR [I \mid -\tilde{C}]$$

$$P = M [I \mid -\tilde{C}]$$

decompose projection matrix P in K,R,C

$$P = [M \mid \mathbf{p}_4] \quad [K, R] = \underbrace{RQ}(M) \quad \tilde{C} = -M^{-1}\mathbf{p}_4$$

RQ matrix
decomposition

Finite Projective Camera – Summary

$$\begin{aligned} P &= KR[I \mid -\tilde{C}] \\ &= M[I \mid -\tilde{C}] \end{aligned} \quad \text{where } P \in \mathbf{R}^{3 \times 4}$$

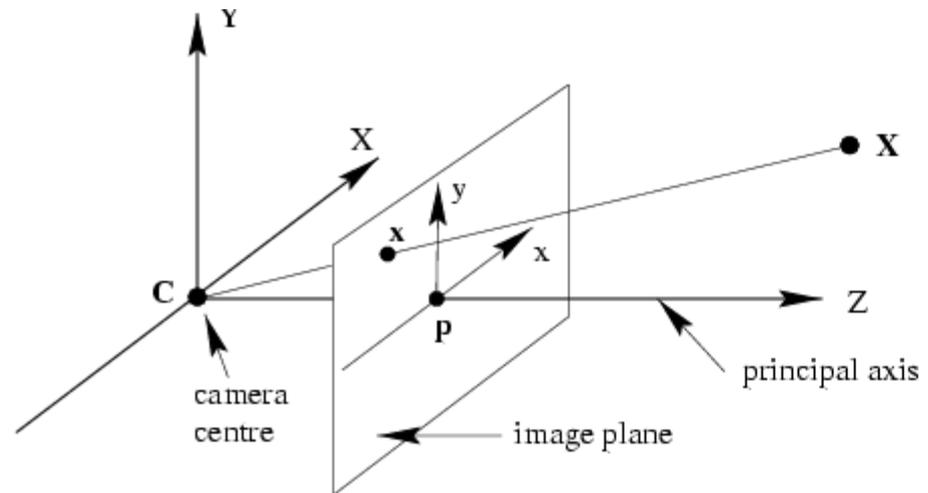
- Camera matrices P are identical with the set of homogeneous 3×4 matrices for which the left 3×3 sub-matrix is non-singular
- $\{\text{finite cameras}\} = \{P \mid \det M \neq 0\} = \{P \mid \text{rank}(M) = 3\}$
- If $\text{rank}(P) = 3$, but $\text{rank}(M) < 3$, then camera at infinity
- if $\text{rank}(P) < 3$ the matrix mapping will be a line or a point and not a plane (not a 2D image)

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Camera Anatomy

- Camera center
- Column vectors
- Principal plane
- Axis plane
- Principal point
- Principal ray



Camera Center

$$PC = 0 \quad P \text{ has a 1D null-space}$$

we will prove that the 4-vector C is the camera center

$$X(\lambda) = \lambda A + (1 - \lambda)C \quad \text{points on a line through } A \text{ and } C$$

$$x = PX = \lambda PA + (1 - \lambda)PC = \lambda PA \quad \text{since } PC = 0$$

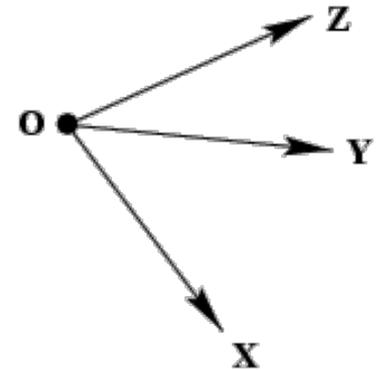
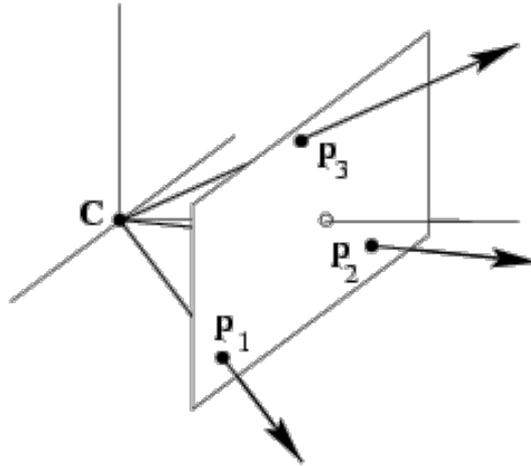
All 3D points on the line are mapped on the same 2D image point, and thus the line is a ray through the camera center

$$\text{Finite cameras:} \quad C = \begin{pmatrix} \tilde{C} \\ 1 \end{pmatrix} = \begin{pmatrix} -M^{-1}\mathbf{p}_4 \\ 1 \end{pmatrix}$$

$$\text{Infinite cameras:} \quad C = \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix}, M\mathbf{d} = 0$$

Column Vectors

$$\mathbf{p}_i \in \mathbf{R}^3, i = 1, \dots, 4$$



Column vectors are the image points which project the axis directions (X,Y,Z) and the origin

Example for the image of the y-axis

$$[\mathbf{p}_2] = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \quad \mathbf{p}_4] \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

\mathbf{p}_4 is the image of the world origin

Row Vectors

$$\mathbf{p}^i \in \mathbf{R}^4, i = 1, \dots, 3$$

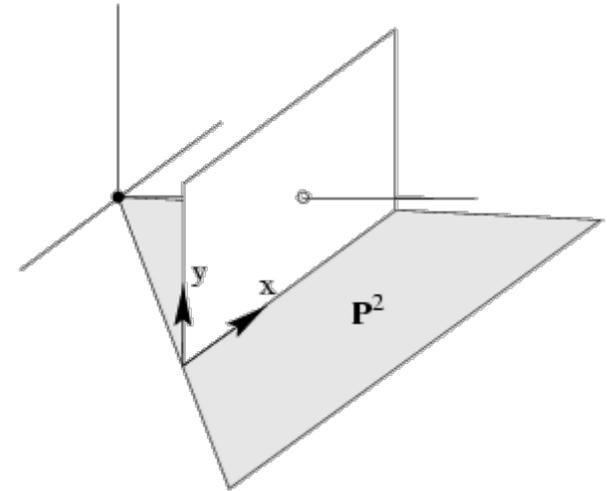
Represent geometrically particular world planes.

$$\begin{aligned}
 \mathbf{P} &= \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix} && \text{row vectors} \\
 &= \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix} && \text{column vectors}
 \end{aligned}$$

Row Vectors of the Projection Matrix

P^1 is defined by the camera center and the line $x=0$ on the image.

P^2 is defined by the camera center and the line $y=0$ on the image.



Example P^2

$$\begin{bmatrix} x \\ 0 \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{1\top} \\ \mathbf{p}^{2\top} \\ \mathbf{p}^{3\top} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

respectively for P^1

$$\mathbf{p}^{2\top} \mathbf{X} = 0$$

$$\mathbf{p}^{2\top} \mathbf{C} = 0$$

Principal Plane

Plane through camera center and parallel to the image plane.

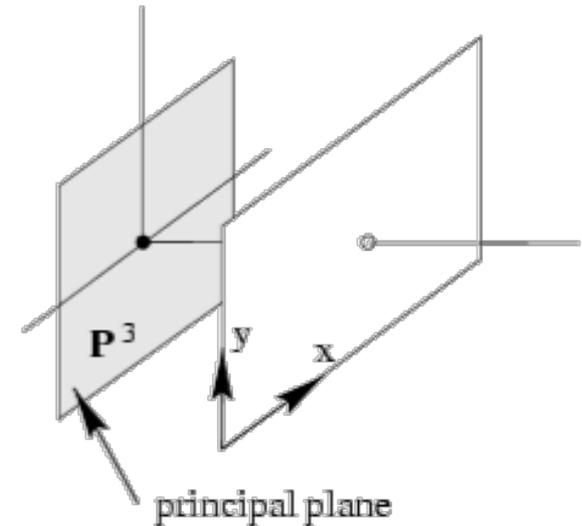
$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

if \mathbf{X} is on the principle plane

$$\mathbf{p}^{3T} \mathbf{X} = 0$$

especially

$$\mathbf{p}^{3T} \mathbf{C} = 0$$

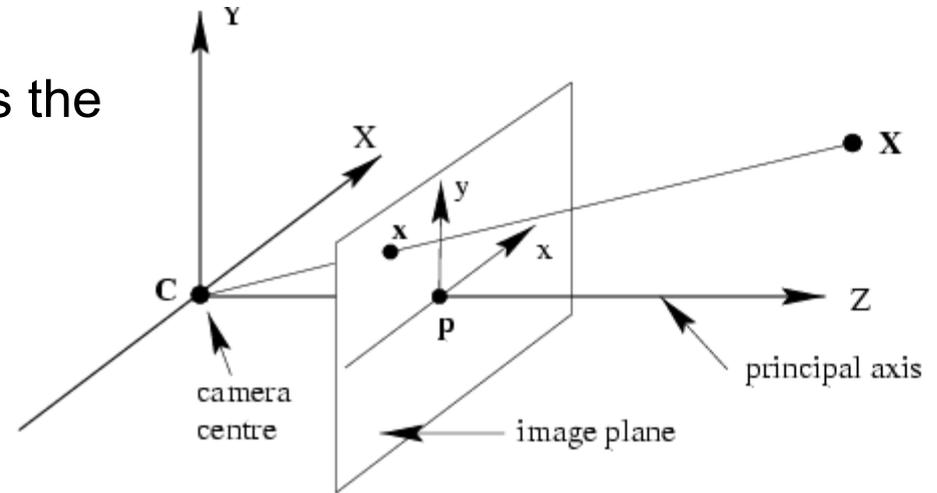


points \mathbf{X} are imaged on the line at infinity

Principal Point

Line through camera center and perpendicular to principal plane is the principal axis.

Intersection of the principal axis with the image plane is the principal point.



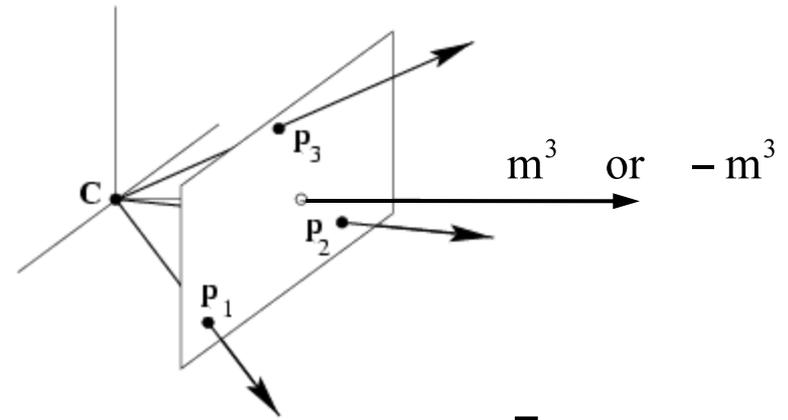
$\hat{\mathbf{P}}^3 = (p_{31} \quad p_{32} \quad p_{33} \quad 0)^T$ normal direction to principal plane

principal point

$\mathbf{X}_0 = \mathbf{P}\hat{\mathbf{P}}^3 = \mathbf{M}\mathbf{m}^3$ where $\mathbf{P} = [\mathbf{M} \mid \mathbf{p}_4]$
and \mathbf{m}^{3T} third row of \mathbf{M}

Principal Axis Vector

Ambiguity that principal axis points towards the front of the camera (positive direction)



$$X = P_{\text{cam}} X_{\text{cam}} = K[I | 0] X_{\text{cam}}$$

$$v = \det(K) k^3 = (0, 0, 1)^T$$

towards the front of the camera

$$P_{\text{cam}} \mapsto k P_{\text{cam}} \quad v \mapsto k^4 v$$

direction unaffected by scaling

$$P = kKR [I | -\tilde{C}] = [M | p_4]$$

since $\det(R) > 0$

$$v = \det(M) m^3$$

Forward Projection

Maps a point in space on the image plane $\mathbb{P}^3 \rightarrow \mathbb{P}^2$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

Vanishing points $\mathbf{D} = (\mathbf{d}^T, 0)^T$

$$\mathbf{x} = \mathbf{P}\mathbf{D} = \left[\mathbf{M} \mid \mathbf{p}_4 \right] \mathbf{D} = \mathbf{M}\mathbf{d}$$

Only \mathbf{M} affects the projection of vanishing points

Back-Projection to Rays

Points on the reconstructed ray

$$PC = 0 \quad \text{camera center } C$$

$$X = P^+ x \quad P^+ = P^T (PP^T)^{-1} \quad PP^+ = I$$

(pseudo-inverse)

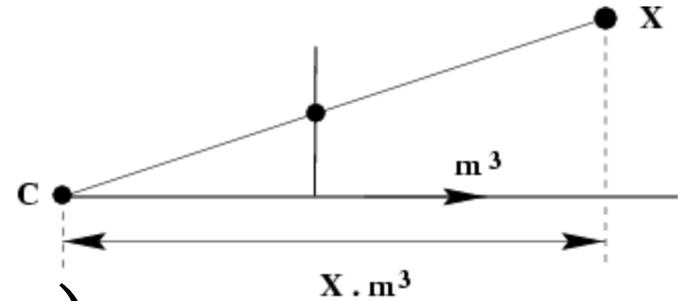
Ray is the line formed by those two points

$$X(\lambda) = P^+ x + \lambda C$$

$$D = \left(\left(M^{-1} x \right)^T, 0 \right)^T \quad \text{intersection of the ray with the plane at infinity}$$

$$X(\mu) = \underbrace{\mu \begin{pmatrix} M^{-1} x \\ 0 \end{pmatrix}}_D + \underbrace{\begin{pmatrix} -M^{-1} p_4 \\ 1 \end{pmatrix}}_C = \begin{pmatrix} M^{-1} (\mu x - p_4) \\ 1 \end{pmatrix}$$

Depth of Points



$$w = P^{3T} X = P^{3T} (X - C) = m^{3T} (\tilde{X} - \tilde{C})$$

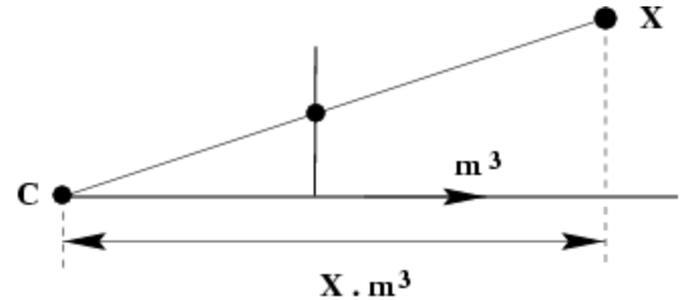
(PC=0) (dot product)

If $\det M > 0; \|m^3\| = 1$,
then m^3 unit vector pointing in positive axis direction

Suppose $P(X, Y, Z, T)^T = w(x, y, 1)^T$. Then

$$\text{depth}(X; P) = \frac{\text{sign}(\det M) w}{T \|m^3\|}$$

Depth of Points: Examples



$$\text{depth}(X; P) = \frac{\text{sign}(\det M)w}{\|m^3\|}$$

$$P(X, Y, Z, 1)^T = w(x, y, 1)^T$$

$$\mathbf{x} = P\mathbf{X}$$

$$\mathbf{x} = P\mathbf{X}$$

$$P = [I | 0]$$

$$P = R[I | -\tilde{C}]$$

$$\text{depth}(X; P) = Z$$

$$\text{depth}(X; P) = R^{3T}(\tilde{X} - \tilde{C})$$

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Camera Matrix Decomposition

Finding the camera center C

$$PC = 0$$

numerically: find right null-space by SVD of P

$$P^{3 \times 4} = U^{3 \times 4} D^{4 \times 4} V^{4 \times 4 T} \rightarrow P^{3 \times 4} V^{4 \times 4} = U^{3 \times 4} D^{4 \times 4} \rightarrow PV^4 = 0$$

Algebraically:

$$X = \det \left(\begin{bmatrix} p_2 & p_3 & p_4 \end{bmatrix} \right) \quad Y = -\det \left(\begin{bmatrix} p_1 & p_3 & p_4 \end{bmatrix} \right)$$

$$Z = \det \left(\begin{bmatrix} p_1 & p_2 & p_4 \end{bmatrix} \right) \quad T = -\det \left(\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \right)$$

where $C = (X, Y, Z, T)^T$

Camera Matrix Decomposition

Finding the camera center C

$$PC = 0$$

Any plane π going through C will be a linear combination of the three planes defined by the rows of P . Therefore:

$$\begin{aligned} \det \left(\begin{bmatrix} \pi & P^T \end{bmatrix} \right) &= 0 \\ &= \pi_1 \det(P_{234}) - \pi_2 \det(P_{134}) + \pi_3 \det(P_{124}) - \pi_4 \det(P_{123}) \end{aligned}$$

$$\begin{aligned} X &= \det \left(\begin{bmatrix} p_2, p_3, p_4 \end{bmatrix} \right) & Y &= -\det \left(\begin{bmatrix} p_1, p_3, p_4 \end{bmatrix} \right) \\ Z &= \det \left(\begin{bmatrix} p_1, p_2, p_4 \end{bmatrix} \right) & T &= -\det \left(\begin{bmatrix} p_1, p_2, p_3 \end{bmatrix} \right) \end{aligned}$$

$$\text{where } C = (X, Y, Z, T)^T$$

Camera Matrix Decomposition

Finding the camera orientation and internal parameters

$$P = [M | -M\tilde{C}] = K[R | -R\tilde{C}] \quad K = \begin{bmatrix} \alpha_x & s & y_0 \\ 0 & \alpha_y & x_0 \\ 0 & 0 & 1 \end{bmatrix}$$

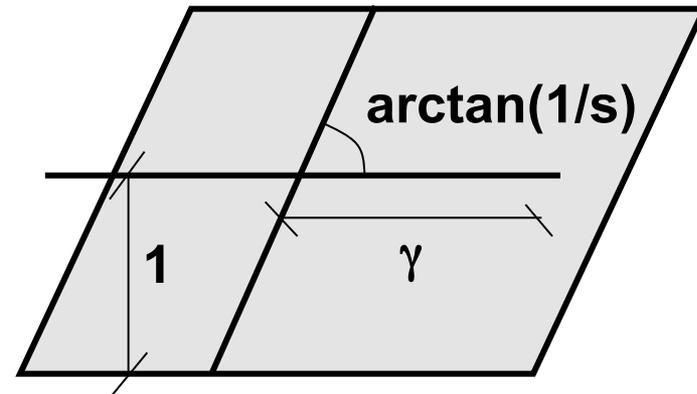
Decompose $M = KR$ using RQ decomposition

$$\square = (\square Q \triangle R)^{-1} = \triangle R^{-1} \square Q^{-1}$$

Ambiguity removed by enforcing positive diagonal entries

When is Skew Non-zero?

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix}$$



for CCD/CMOS, always $s=0$

Image from image, $s \neq 0$ possible
(non coinciding principal axis)

resulting camera: HP
where H is a 3×3 homography

Euclidean vs. Projective Spaces

General projective interpretation

$$P = \left[\begin{array}{c} 3 \times 3 \text{ homography} \\ 4 \times 4 \text{ homography} \end{array} \right] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Meaningful decomposition in K, R, t requires Euclidean image and space
- Camera center is still valid in projective space
- Principal plane requires affine image and space
- Principal ray requires affine image and Euclidean space

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Cameras at Infinity

Cameras with their center lying at infinity

$$P \begin{bmatrix} d \\ 0 \end{bmatrix} = 0 \Rightarrow \det M = 0 \quad M \text{ is singular}$$

Two types of cameras at infinity:
Affine and non-affine cameras

Affine Cameras

Definition:

An affine camera is a camera with a camera matrix P in which the last row p^{3T} is of the form $(0,0,0,1)^T$.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Points at infinity are mapped to points at infinity

Parallel Projections

$$P_{\infty} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

canonical representation

$$K = \begin{bmatrix} K_{2 \times 2} & 0 \\ 0 & 1 \end{bmatrix}$$

calibration matrix

principal point is not defined

Hierarchy of Affine Cameras

$$\mathbf{P}_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{dropping the z-coordinate}$$

orthographic projection

$$\mathbf{P}_\infty = \begin{bmatrix} \mathbf{r}^{1T} & t_1 \\ \mathbf{r}^{1T} & t_2 \\ 0 & 1 \end{bmatrix} \quad (5\text{dof})$$

Hierarchy of Affine Cameras

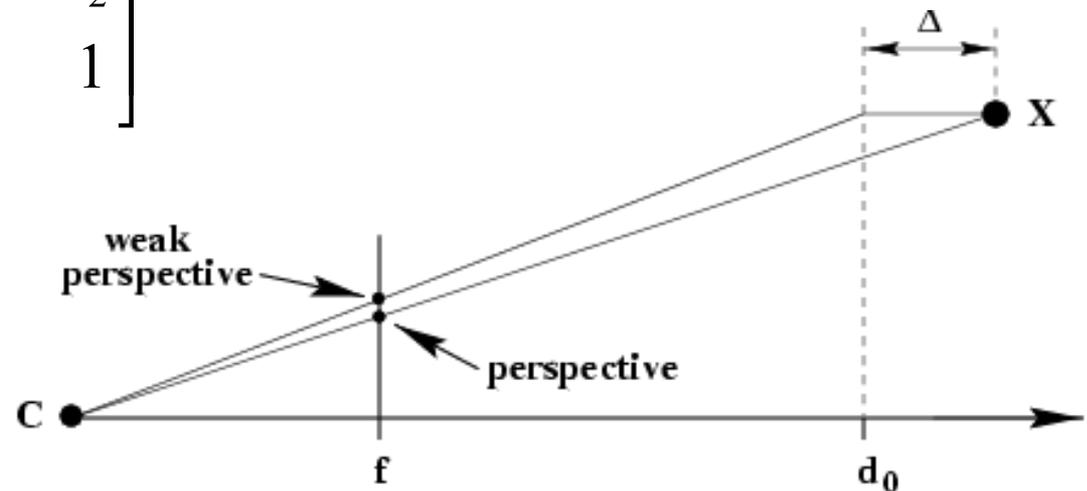
scaled orthographic projection

$$\mathbf{P}_\infty = \begin{bmatrix} k & & & \\ & k & & \\ & & & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}^{1T} & t_1 \\ \mathbf{r}^{1T} & t_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{r}^{1T} & t_1 \\ \mathbf{r}^{1T} & t_2 \\ 0 & 1/k \end{bmatrix} \quad (6\text{dof})$$

Hierarchy of Affine Cameras

weak perspective projection

$$P_{\infty} = \begin{bmatrix} \alpha_x & & & \\ & \alpha_y & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}^{1T} & t_1 \\ \mathbf{r}^{1T} & t_2 \\ 0 & 1 \end{bmatrix} \quad (7\text{dof})$$



Hierarchy of Affine Cameras

Affine camera

$$\mathbf{P}_A = \begin{bmatrix} \alpha_x & s & & \\ & \alpha_y & & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}^{1T} & t_1 \\ \mathbf{r}^{1T} & t_2 \\ 0 & 1 \end{bmatrix} \quad (8\text{dof})$$

$$\mathbf{P}_A = \begin{bmatrix} m_{11} & m_{12} & m_{13} & t_1 \\ m_{21} & m_{22} & m_{23} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

full generality of an affine camera

$$\mathbf{P}_A = \begin{bmatrix} 3 \times 3 \text{ affine} & \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ & \end{bmatrix} \begin{bmatrix} 4 \times 4 \text{ affine} \end{bmatrix}$$

- Affine camera is a projective camera with principal plane at infinity
- Affine camera maps parallel world lines to parallel image lines
- No center of projection, but direction of projection $\mathbf{P}_A \mathbf{D} = 0$

General Camera at Infinity

$$P = [M | p_4] \quad M \text{ is singular, but last row not zero}$$

- Camera center is on plane at infinity
- Principal plane is not plane at infinity
- Images of points at infinity are in general not mapped to infinity on the image plane

Summary Camera Models

- Photometric and radiometric properties of a camera
- Geometric parameters of a finite camera
- Projective cameras
 - Camera anatomy (camera center, principle plane, principle point, and principle axis)
 - Camera matrix decomposition (camera center, orientation, and intrinsic parameter)
- Cameras at infinity
 - Affine cameras
 - Non-affine cameras

Literature on Camera Models

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