

# Fusion of Clothoid Segments for a More Accurate and Updated Prediction of the Road Geometry

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**Abstract**—A basic step in the development of advanced driver assistance systems is the perception and interpretation of information on the vehicle environment. In many cases, the upcoming road geometry in front of the own vehicle is of particular interest. Although a prediction of the course of the road can be provided based on map data, the accuracy is not sufficient for most driver assistance systems. The goal of this work is to achieve a more accurate and updated prediction of the road geometry at close range. Therefore, the map data is combined with the information from an existing vision-based lane detection system. Since the shape of detected road lines as well as the map data are represented by clothoid curves, the emphasis of this work is placed on the calculation, combination, and connection of clothoid segments.

## I. INTRODUCTION

Many advanced driver assistance systems are in need of detailed information on the road geometry to improve their functionality. The problem is that this information is usually not available. The available map data is inaccurate and information on the exact position and orientation of the vehicle is missing. In particular on less important streets the map data includes fewer details and may not describe the exact course of the road. Also the accuracy of the vehicle's position provided by a GPS receiver is only about 10-15 meters [1], so that the exact position on the map is not known. The mentioned constraints make the prediction of the upcoming road geometry less precise. However, a video-based lane detection system may provide more detailed information.

In this work, we want to achieve a more accurate and updated prediction of the road geometry at close range. Therefore, the information from an existing vision-based lane detection system is fused with the available map data. The developed methods should not only be applied to well-defined situations on highways, but also, and in particular, to the urban environment. There are numerous difficulties in inner-city situations. Road layouts may be complex, map data is often less precise, and lane markings are missing or unrecognizable. Therefore we developed an algorithm which can handle all these difficulties.

Since the information on the road geometry is represented by clothoid segments, this work focuses on the calculation, combination, and connection of clothoid segments in complex urban regions. The clothoid or Euler spiral is a special type of curve that is used in road construction as well as in many works on lane detection and road modeling. In [2],

for example, a 3D road model is presented that is based on clothoids in the horizontal plane. A clothoid model in combination with a particle filter is used in [3] to detect the course of the road.

In the next sections, the definition of clothoids, a short description of the available map data, and the characteristics of the lane detection system are given. Afterwards, the necessary steps for the fusion of detected lines and the map data are presented: First of all, we show that it is possible to fuse clothoids by averaging the curvature parameters. Then we combine the characteristics of detected lane markings by calculating approximated parallel clothoids and fusing clothoid segments of different lengths. The next step is the fusion of the combined segment with map data. The result is a fused segment with higher accuracy that is connected to the original map data after optimizing its length in the last step.

## II. BASICS

In this section the basics of clothoids and the available sensor systems are introduced.

### A. Clothoids

Clothoids are curves with constant curvature change. For a clothoid segment, its curvature  $c$  starts at  $c_0$  and varies linearly with arc length  $l$ . At a certain point of the clothoid after length  $l$  the curvature is:

$$c(l) = c_0 + c_1 \cdot l. \quad (1)$$

The three parameters of a clothoid segment are the initial curvature  $c_0$ , the curvature change  $c_1$ , and the length of the segment  $l_s$ .

Since clothoids cannot be evaluated in closed form, clothoid segments are often approximated by the following polynomial. The initial lateral offset and heading direction are denoted as  $y_0$  and  $\tau_0$  respectively. According to [2], the polynomial model is sufficiently precise as long as the change in heading direction, the tangent angle  $\tau$ , of the clothoid segment is less than  $15^\circ$ :

$$y \approx y_0 + \tau_0 \cdot l + \frac{1}{2}c_0 \cdot l^2 + \frac{1}{6}c_1 \cdot l^3, \quad (2)$$

$$\tau = \tau_0 + c_0 \cdot l_s + \frac{1}{2}c_1 \cdot l_s^2. \quad (3)$$

## B. Incoming Data

To achieve a more accurate and updated course prediction we fuse map data and road markings detected by a given video-based lane detection system:

The available map data describes the network of roads by the positions of points on the road relative to the own vehicle as well as their curvatures. Predictive information on the road geometry is provided by clothoid segments. The representation of the road by clothoid segments is exploited in this work for the fusion with detected lane markings. However, the clothoid segments are often not as precise as one would expect.

A lane detection camera is available that contains an embedded software module to detect and estimate the position and shapes of lane markings in the video. The system represents each detected line by a single clothoid segment. The two lines that describe the own lane are also called ego lines. The results in different weather conditions are relatively good, and ego lines as well as continuous and dashed lines are categorized. In tight curves with a small radius, the lane detection system often fails. In the next section, detected ego lines are used to correct the map data.

## III. FUSING DETECTED LINES AND MAP DATA

The fusion of map data and detected lane markings is one way to get an improved prediction of the road geometry. This involves the approximation of parallel clothoids as well as the fusion of different clothoid segments. In addition, the fused segments are connected again to the map data using an optimization algorithm that ensures a continuous change of curvature. The result is a more accurate and updated estimation of the road geometry.

### A. Fusion of Clothoids

This section discusses the possible approaches for fusing two or more clothoid segments to obtain an averaged segment that should lie between the initial curves. Although it seems convenient to simply average the clothoid parameters, the characteristics and effects of the parameters should be analyzed first. In [4] the clothoid parameter sloshing is explained. It is possible to change  $c_0$  and  $c_1$  in a way that the resulting shape of the curve hardly changes. Figure 1 illustrates this effect. The first two clothoids have an initial radius of curvature of 1000 meters and 250 meters, but for the next 50 meters they have a similar shape. The third clothoid has only a different initial curvature compared to the first clothoid, but its shape differs significantly. For this reason, the parameters  $c_0$  and  $c_1$  of clothoid segments have to be fused at the same time and should not be changed independently.

There is a simple example provided in [4] that compares two different methods of fusing clothoid segments. The first one averages the coefficients  $y_0$ ,  $\tau_0$ ,  $c_0$ , and  $c_1$  of the clothoid model. The alternative is to calculate the lateral offsets  $y$  of the clothoid at certain distances  $x$  and to average these values. Although the example shows that the results obtained by averaging clothoid parameters may be ambiguous, both

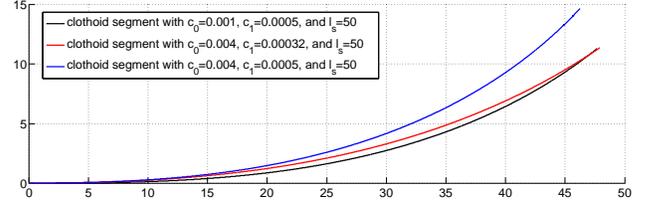


Fig. 1. Influence of clothoid parameters on the curve shape

methods can produce the same results. This is shown by a simple correction of the averaging method:

There are four different sensors in the example that are used to estimate the road geometry in front of the vehicle: a yaw rate sensor, map data, lane detection and a radar sensor. Each sensor provides noisy measurements for a subset of the coefficients of the clothoid model. It is assumed that the true values for the clothoid segment that describes the road geometry are  $y_0 = 0.0$ ,  $\tau_0 = 0.0$ ,  $c_0 = 1/300$ , and  $c_1 = -0.00002$ . In the original description of the example,  $y_0$  is indicated by the letter  $t$  and  $\tau_0$  by the letter  $s$ . The next assumption is that the sensors provide measurements for the lateral offsets at the distance  $x = 30, 40, \dots, 120$  with a standard deviation of  $\sigma = 0.1$ . The following figures show the results for 1000 trials of fusing the sensor measurements to obtain an averaged clothoid. At first, the results for the second method are shown in Fig. 2. The measured lateral offsets are averaged, and a fused clothoid is determined using the method of least squares:

$$\bar{y}_i = (y_{i,yawrate} + y_{i,map} + y_{i,lanedetection} + y_{i,radar})/4, i = 30, 40, \dots, 120. \quad (4)$$

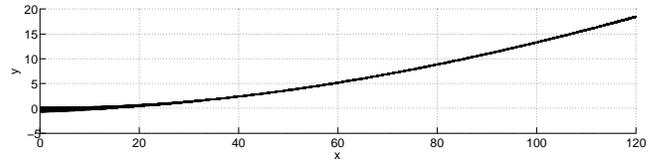


Fig. 2. Road geometry estimation by fusing lateral offsets

The other method is to fuse the clothoid parameters and not the lateral offsets. Therefore, a clothoid is calculated for each sensor based on the virtual measurements. The resulting coefficients of the clothoid model are then averaged to obtain a fused clothoid segment:

$$\bar{y}_0 = (0 + 0 + y_{0,lanedetection} + y_{0,radar})/4, \quad (5)$$

$$\bar{\tau}_0 = (0 + 0 + \tau_{0,lanedetection} + \tau_{0,radar})/4, \quad (6)$$

$$\bar{c}_0 = (c_{0,yawrate} + c_{0,map} + c_{0,lanedetection} + c_{0,radar})/4, \quad (7)$$

$$\bar{c}_1 = (0 + c_{1,map} + c_{1,lanedetection} + 0)/4. \quad (8)$$

The output of Fig. 3 leads to the conclusion that, in contrast to [4], there is no problem in fusing the coefficients

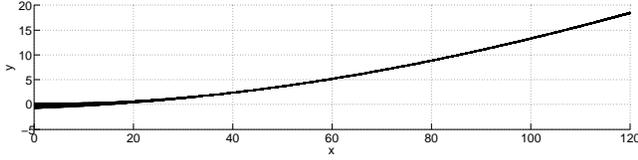


Fig. 3. Road geometry estimation by correctly fusing clothoid parameters

of the clothoid model as long as the clothoid segments are comparable. In [5] it is even noted that the fusion of clothoid parameters is in many cases more stable than the fusion of lateral offsets. Problems arise mainly when clothoid segments have different lengths or the initial lateral offset is high. Solutions for these cases are presented in the next sections.

### B. Approximation of Parallel Clothoid Segments

When estimating the road geometry in front of the vehicle, the clothoid segments that describe the shape of the road should start right in front of the vehicle. Usually the offset of a detected line is not 0. Therefore, a parallel clothoid is needed (cf. [6]).

For a clothoid segment with a curvature change other than zero, the radius of curvature changes with arc length. So for a parallel curve with distance  $\Delta r$ , four different conditions have to be met, whereas  $c_2$  is the curvature at the end of the segment:

$$\tilde{c}_0 = \frac{1}{\frac{1}{c_0} + \Delta r}, \tilde{c}_2 = \frac{1}{\frac{1}{c_2} + \Delta r}, \tilde{\tau} = \tau, \tilde{l}_s = l_s + \Delta r \cdot \tau. \quad (9)$$

These four conditions are required for an exactly parallel clothoid segment, but it is not possible to satisfy all of them at the same time unless  $c_0$  and  $c_2$  are equal or  $\tau$  is zero. The reason is that a clothoid segment is defined by three parameters and if three conditions are selected, the fourth condition cannot be met in general. Fig. 4 shows the results for approximated parallel clothoid segments when ignoring the length  $l_s$ , the end curvature  $c_2$ , or the change in tangent angle  $\tau$ . The error at the end of the approximated clothoid segments increases as the curvature change  $c_1$  of the original segment increases. Three selected examples are listed in table I:

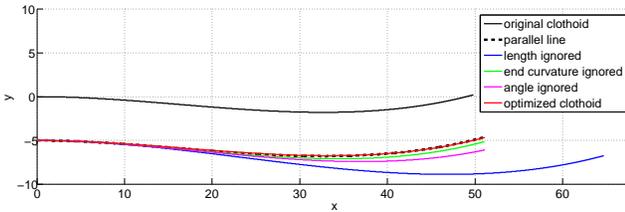


Fig. 4. Errors when approximating a parallel clothoid segment

The examples also show the results for optimized clothoid segments that are very close to the actual parallel line. The idea is to minimize the square errors between the four

TABLE I

ERRORS AT THE END OF APPROXIMATED PARALLEL SEGMENTS

Clothoid segment parameters	$\Delta r$	ignored parameter	error distance
$c_0 = -0.01$ $c_1 = 0.00061$ $l_s = 50$	5	$\tilde{\tau}$ $\tilde{l}_s$ $\tilde{c}_2$ optimized	1.45 13.79 0.51 0.04
$c_0 = 0.0$ $c_1 = 0.00021$ $l_s = 50$	5	$\tilde{\tau}$ $\tilde{l}_s$ $\tilde{c}_2$ optimized	0.17 1.31 0.06 0.00
$c_0 = 0.005$ $c_1 = 0.000001$ $l_s = 50$	5	$\tilde{\tau}$ $\tilde{l}_s$ $\tilde{c}_2$ optimized	0.01 0.07 0.00 0.00

expected values  $\tilde{c}_0$ ,  $\tilde{c}_2$ ,  $\tilde{\tau}$ , and  $\tilde{l}_s$  and the selected parameter values for a parallel line. Different least-squares methods were applied that all lead to similar results:  $\tilde{l}_s$  should not be changed, and  $\tilde{c}_0$  and  $\tilde{c}_2$  have to be adapted in a way that the resulting change in tangent angle equals the expected value  $\tilde{\tau}$ . This means that if only the curvature parameters are changed, the approximated clothoid segment is still close to a parallel line.

### C. Fusion of Clothoid Segments of Different Length

The aim of this chapter is to obtain a centered clothoid segment that can be fused later with the road geometry provided by map data. Figure 5 shows the necessary steps to fuse the two clothoid segments of the ego lane: At first, parallel segments are calculated so that both segments have their origin in front of the own vehicle. The initial heading direction  $\tau_0$  may be averaged, but should be the same for both segments. Since the map data provides no heading angle, it is also not required at this step for the centered clothoid segment. After all, only the two curvature parameters  $c_0$  and  $c_1$  are left and have to be averaged:

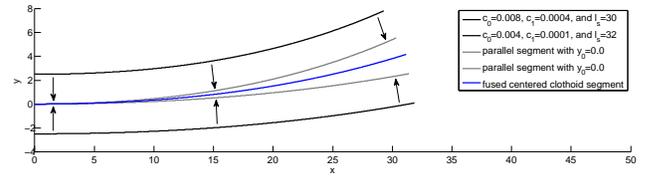


Fig. 5. Fusion of clothoid segments

The result is a fused clothoid segment that lies in the center of the two original segments. Although averaging clothoid parameters works perfectly in this case, the results may be less satisfactory if the lengths of the two original clothoid segments differ more. Fig. 6 demonstrates the problem for the case that the right clothoid segment is longer. If the curvature change is averaged and the centered clothoid segment should have length  $l_s = 50$ , the shorter segment is implicitly extrapolated to the same length. However, if the lane marking is only detected up to a certain distance, it is

not a good idea to assume that the curvature change remains the same beyond this distance. The lane detection system determined the shape of the line only up to this distance with suitable clothoid parameters. If the line is extended, these parameters usually have to be adapted. It is, on the contrary, the longer line that provides more information and that should be weighted more.

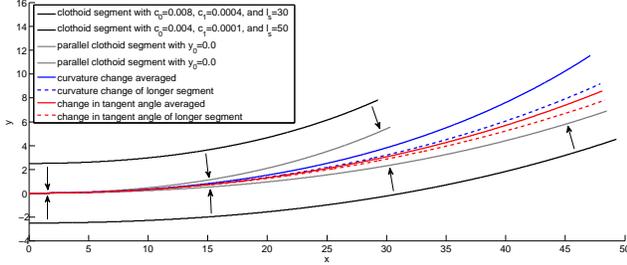


Fig. 6. Fusion of clothoid segments of different length

The question is how to perform the weighting. The calculation of the initial curvature  $c_0$  should not be changed, because both lane markings are detected at short range and may provide suitable values. Also for the fusion of connected segments in the next section, the initial curvature of following segments will be given and cannot be changed. For these reasons, only the curvature change  $c_1$  can be weighted. In figure 6, the dashed blue line shows the result if  $c_1$  is selected to be equal to the curvature change of the longer line, but the distance between the longer line and the fused segment still increases. The reason is again that the curvature parameters  $c_0$  and  $c_1$  are not independent and both influence the entire shape of the clothoid segment.

Therefore, the idea is to consider the change in heading direction rather than the curvature change. By rearranging the equation for the sum of the initial curvature and the curvature at the end of the segment, it can be seen that the change in tangent angle per meter equals the average curvature along the clothoid segment  $(c_0 + c_2)/2$ :

$$c_0 + c_2 = c_0 + (c_0 + c_1 l_s) = \frac{1}{l_s} (2c_0 l_s + c_1 l_s^2) = \frac{2\tau}{l_s}, \quad (10)$$

This means that averaging the changes in tangent angle per meter produces the same results as averaging the average curvature of clothoid segments. Moreover, if the change in tangent angle per meter of a clothoid segment is weighted according to the segment's length, it is also possible to average the change in tangent angle  $\tau$ . Finally, if the average change in tangent angle is calculated, the curvature change of the fused clothoid segment can be determined:

$$c_1 = \frac{c_2 - c_0}{l_s} = \frac{2(\frac{c_0 + c_2}{2}) - 2c_0}{l_s} = \frac{2(\frac{\tau}{l_s}) - 2c_0}{l_s}. \quad (11)$$

The red line in figure 6 shows the fused clothoid segment if the change in heading direction per meter is averaged. It is again a bit closer to the longer segment. In addition,

the dashed red segment is the result if only the tangent angle of the longer line is considered. Obviously, this method offers the possibility to weight the parameters of the original clothoid segments in a way that the fused segment follows the longer line. If the tangent angles are weighted by the length of the original segments, satisfying results can be achieved in both cases: For segments of similar length, the fused segment still lies in the center, and for segments of different length, the longer segment is weighted more.

For the fusion of the clothoid segments that describe the left and right lane markings of the own traffic lane, the following formula is used to obtain a weighted average of the change in tangent angle. The higher the exponent  $w$ , the closer is the fused segment to the longer segment.

$$\bar{\tau} = \frac{\tau_{left} \cdot l_{s,left}^w + \tau_{right} \cdot l_{s,right}^w}{l_{s,left}^w + l_{s,right}^w} \quad (12)$$

#### D. Fusion of Connected Segments

The next step is the fusion of the centered clothoid segment that is based on lane detection with the clothoid segments that are based on map data. The aim is to update the first segments of the predictive tree with the information obtained from the lane detection system. Therefore, the methods that were introduced in the previous sections are applied. For comparable segments, the curvature parameters  $c_0$  and  $c_1$  are averaged and weighted. Otherwise, the changes in tangent angle are averaged. The fused segments should provide a better estimate of the road geometry and replace the original segments. In the next section, the corrected segments are connected to the original tree again.

#### E. Optimized Connection of Clothoids

A final step is required to smoothly connect the fused clothoid segments from the last section with the map data. In figure 7, the red line represents a fused clothoid segment that has to be connected to the map segment shown in black. If there are more segments in the relevant range, the last segment of the fused curve has to be connected to a suitable segment of the map data.

The purpose of the two connecting segments is not to define a curve that turns back to the map data segments, but to gradually change the curvature and the tangent angle from the end of the fused segment to the curvature and tangent angle at a certain position of the clothoid segment based on map data. Since the length of the fused segment depends on the lane detection system and varies with time, the connection point on the map data segments is selected to be at a fixed distance. So the lengths of the connecting segments vary as well, but the rest of the tree after the connection point remains unchanged. In Fig. 7, the connection point is selected at the distance  $l_f = 60$ . After determining the connecting segments, the original tree is replaced up to this distance. As a consequence of this, the rest of the tree may be translated by a small offset. This offset cannot be avoided if the curvature of the connecting segments should not get too high. The positive and negative effects of the additional offset will be discussed in the evaluation.

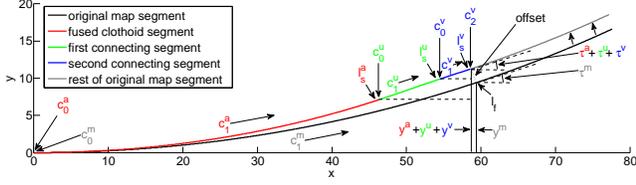


Fig. 7. Connecting clothoid segments for updated map data

The reason for using two connecting segments is the number of degrees of freedom. Each clothoid segments has three parameters: The initial curvature  $c_0$ , the segment length  $l_s$ , and the curvature change  $c_1$  or, alternatively, the resulting end curvature  $c_2$ . For the connection with the map data segments, four conditions must be met: The length of the connection has to be equal to the difference between the length of the fused segment and the distance to the connection point. The start curvature must be equal to the end curvature of the fused segment, and also the end curvature and the tangent angle at the end must be equal to the curvature and tangent angle of the map segment at the connection point. A single clothoid segment cannot fulfill these requirements, but with two clothoid segments the connection can be created as shown in Fig. 7. While the sum of the lengths of both connecting segments is set, the length of one of the segments can be chosen, for example  $l_s^u$ . Then it is possible to determine the other five parameters. For the curvature between the two segments  $c_0^v = c_2^u$ , a formula is derived with the help of equation (10):

$$l_s^v = l_f - l_s^a - l_s^u, \quad (13)$$

$$c_0^u = c_2^a, \quad (14)$$

$$c_2^v = c_0^m + l_f \cdot c_1^m, \quad (15)$$

$$\tau^m - \tau^a = \tau^u + \tau^v = \frac{c_0^u + c_2^u}{2} \cdot l_s^u + \frac{c_0^v + c_2^v}{2} \cdot l_s^v \quad (16)$$

$$\Rightarrow c_0^v = c_2^u = \frac{2(\tau^m - \tau^a) - l_s^u \cdot c_0^m - l_s^v \cdot c_2^m}{l_s^u + l_s^v}. \quad (17)$$

By using these equations, the fused clothoid segment can replace the first part of the map data, and the curvature of the connecting segments changes continuously. The only disadvantage is the offset at the connection point that increases as the difference between the curvatures of the fused segment and the map segments increases. To reduce this offset, the lengths of the fused segment and the first connecting segment may be reduced. In the following, improved values  $\tilde{l}_s^a$  and  $\tilde{l}_s^u$  for the lengths of these two segments are determined. Important criteria for the optimization are the changes in length, the curvature change of the first connecting segment, and the offset at the connection point. Other approaches have shown that the length of the second connecting segment has nearly no effect on the resulting offset and that the curvature change of the second segment stays relatively small. The offset can only be approximated by using the polynomial model for clothoids (2), because an explicit expression is needed for the optimization. The approximation

has proven to be satisfactory in this case, since only the relative difference of the lateral offsets is required.

$$r_1 = \tilde{l}_s^a - l_s^a \quad (18)$$

$$r_2 = \tilde{l}_s^u - l_s^u \quad (19)$$

$$r_3 = c_1^u = \frac{c_2^u - c_0^u}{\tilde{l}_s^u} \quad (20)$$

$$r_4 = y^a + y^u + y^v - y^m \quad (21)$$

The calculations of the residuals or errors  $r_3$  and  $r_4$  need to be further modified by the known equations for clothoid segments. They have to depend only on the parameters  $\tilde{l}_s^1$  and  $\tilde{l}_s^2$  as well as on the fixed values  $c_0^m$ ,  $c_1^m$ ,  $l_f$ ,  $c_0^a$ , and  $c_1^a$ . Afterwards, the squared errors  $(r_i)^2$  are weighted and the sum  $s$  is minimized using a method for nonlinear least-squares problems.

$$s = w_1 \cdot (r_1)^2 + w_2 \cdot (r_2)^2 + w_3 \cdot (r_3)^2 + w_4 \cdot (r_4)^2 \quad (22)$$

For the implementation, we chose small values  $w_3$  for the curvature change and high values  $w_4$  for the desire to minimize the offset. One step of the Gauß-Newton algorithm, that is described in detail in [7], is used to obtain suitable values for the lengths  $l_s^u$  and  $l_s^v$ . As a result of this, the offset and the maximal curvature of the updated map data are reduced. Moreover, the corrected tree is less sensitive to variations in the fused clothoid segments.

#### IV. RESULTS

The benefit of fusing map data with detected lane markings becomes especially obvious on urban and country roads with relatively high curvatures. The increased accuracy of the updated map data is shown in Fig. 8 while driving in a curve on a rural road. The shape of the detected lane markings is adopted to some degree, the heading direction is added, and the two connecting clothoid segments are inserted. The connection point is selected at a distance of 120 meters. The fused segment is shorter than the detected line on the right due to the optimization step.

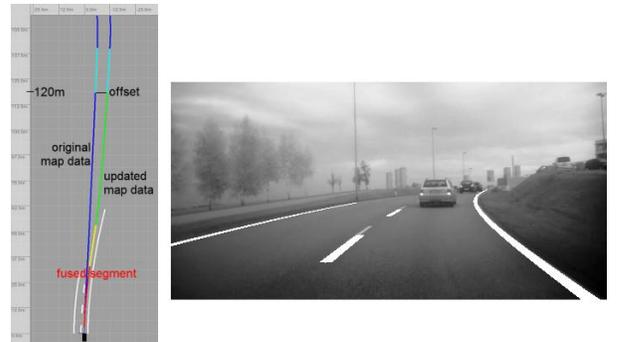


Fig. 8. Updated estimate of the road geometry

To validate the achieved course prediction by fusing map data and lane markings, we compare it to the driven trajectory of the test vehicle within the same street section. Therefore

we calculate the lateral deviations between the trajectory and the course prediction. We demonstrate the improvements of our approach by comparing the lateral distances of the map data and the fused course prediction to the trajectory. In Fig. 9 and 10 the deviations of predictions on a multilane road and within a curve are illustrated. By comparing the values of the deviations to the trajectory it is obvious, that the fused prediction achieves smaller distances to the trajectory of the test vehicle than the original map data.

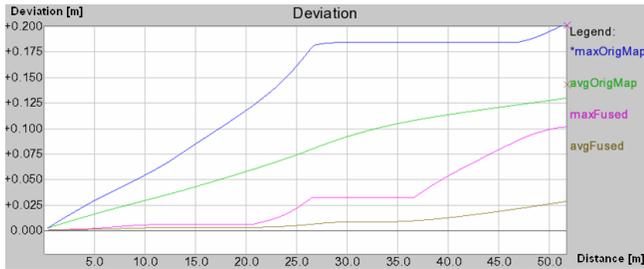


Fig. 9. Deviations of the predictions on a multilane road

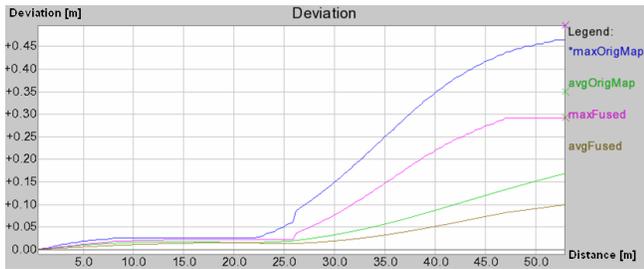


Fig. 10. Deviations of the predictions within a curve

If the additional offset at the connection point improves or worsens the prediction of the road geometry, depends mainly on the quality and length of the detected lanes. If a curve is still not visible, it may be missing in the prediction of the course of the road. Therefore, the aim is to keep the additional offset small. That is achieved by the presented optimization step for the lengths of the last fused segment and the first connecting segment. The average offsets with and without optimization on different types of roads are listed in table II. In particular, on urban roads and country roads, the offset is considerably reduced.

TABLE II  
REDUCED OFFSETS

type of road	duration	average offset before optimization	average offset after optimization
Urban road	1:40 min	1.71 m	0.97 m
Urban road	2:05 min	1.40 m	0.92 m
Urban road	2:42 min	0.77 m	0.56 m
Country road	2:30 min	1.14 m	0.84 m
Country road	2:05 min	1.99 m	1.49 m
Country road	1:10 min	1.00 m	0.81 m

In certain situations, the additional offset may have a negative effect on the corrected map data. Beyond the range of

the detected lane markings an overshooting of the connecting segments can occur, so that the updated course is erroneously shifted from the original map data. This may be observed for example in Fig. 11. However, with the optimized connection, the fused segment is shortened and the lateral shift can be reduced. In addition, the effects of sudden changes in the shape of the fused clothoid segment are reduced as well. If the curvatures of the detected lane markings and the map data differ too much, the shortened segment has less influence on the updated map data.

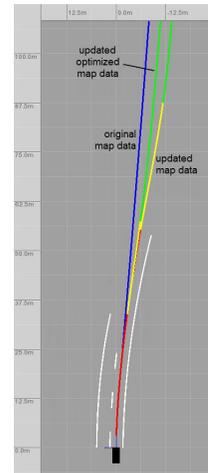


Fig. 11. Original tree, updated tree, and optimized tree

## V. CONCLUSIONS

In this paper, several techniques have been proposed to combine the information provided by the lane detection system with map data to obtain an updated estimate of the road geometry. Methods for the fusion of clothoid segments and the approximation of parallel segments have been introduced that can be used to calculate the required clothoid segments. Finally, the fused segments are smoothly connected to the original map data to create a more accurate and updated prediction of the road geometry.

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